Quantum Physics

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Putting down her cup of tea, she asked in a timid voice, 'Is light made of waves, or is it made of particles?' — Alice's Adventures in Wonderland

welcome to the fun house

Ever since her last science class, Alice has been deeply puzzled by something, and she hoped one of her new acquaintances might straighten out the confusion. Putting down her cup of tea, she asked in a timid voice, 'Is light made of waves, or is it made of particles?'

'Yes, exactly so,' replied the Mad Hatter.

Somewhat irritated, Alice asked in a more forceful voice, 'What kind of answer is that? I will repeat my questions: Is light particles or is it waves?'

'That's right,' said the Mad Hatter.

1. classical theories
2. photon theory
3. wave-particle duality
4. quantisation of energy levels
5. band theory

particle model

substances are made of particles (atoms/molecules) described by Newtonian mechanics

macroscopic behaviour of matter is explained by looking into microscopic behaviour of particles

area of physics	particle description	macroscopic phenomena
electricity	flow of electrons	electric current
gases	kinetic theory	pressure, temperature
solids	crystalline materials	elasticity, modulus
radioactivity	nuclear model	radioactivity, fission, fusion
chemistry	atomic structure	chemical reactions

wave model

characteristic properties of waves: interference and diffraction

interference: two waves superimpose to form a resultant wave of greater or lower amplitude

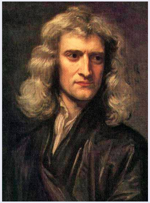
diffraction: effect of waves when encountering an obstacle

phenomena	varying quantity
sound	pressure, density
electromagnetic waves	electric and magnetic fields
waves on strings	displacement

particle theory

particle theory of light (Issac Newton, 1671)

light rays is comprised of a stream of massless particles (corpuscles)



- explains straight-line propagation of light
- explains reflection and refraction of light
- explains colours of light (white light contains different colour particles)

wave theory

wave theory of light (Christian Huygens, 1678)

light is a wave that transfers energy within a medium (aether)

- follows laws of reflection and refraction
- explains interference and diffraction (Thomas Young's double slit experiment, Poisson spot experiment)

most scientists believed light travelled through space as a wave

electromagnetic theory

electromagnetic theory (James Maxwell)
a set of four equations to fully describe behaviour of EM fields

light is an electromagnetic wave

electric and magnetic fields travel through space in the form of waves at speed of light (does not require a medium)

proved by Heinrich Hertz in 1887 by transmitting and receiving radio waves

1. classical theories
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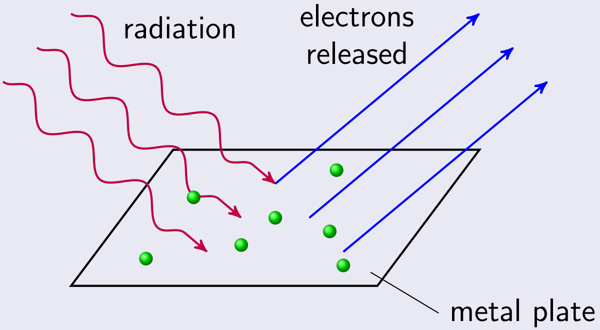
photoelectric effect

when electromagnetic radiation (eg. from a mercury lamp) is incident on metal plate (eg. zinc plate), a current is observed



photoelectric effect

conduction electrons – not tightly held, free to move around

when EM radiation strikes metal, electrons break free

this process is called photoelectric effect (or photoemission)

these ejected electrons are called photoelectrons

first observed in 1887 by Heinrich Hertz, who found electrodes illuminated with ultraviolet light create electric sparks easily

characteristic properties of photoelectric effect

- exists a minimum threshold frequency f_0, when $f < f_0$, no electrons released
- immediate emission of electrons happens as light is turned on
- low intensity light is effective
- increasing intensity has no effect on energies of electrons
- increasing intensity of incident light increases number of photoelectrons emitted
- increasing radiation frequency increases electron energies

breakdown of wave theory

wave theory of light fails to explain these properties

according to wave model of light:

- greater intensity mean higher energy, easier to release electrons
- need very intense light to have immediate effect
- low-frequency light should work, electrons could absorb energies gradually

some new ideas are here needed!

birth of quantum theory

photoelectric effect was first explained by Albert Einstein in 1905

Albert Einstein was awarded the Nobel physics Prize in 1921 for "*his discovery of the law of the photoelectric effect*"

Einstein's revolutionary idea: light energy is not continuous but carried in discrete packets

all electromagnetic radiation consists of photons

Einstein relation

a photon is a packet (quanta) of electromagnetic energy

energy of one photon: $\boxed{E = hf}$ known as Einstein relation

Planck constant: $h = 6.63 \times 10^{-34}$ J · s

Example

γ-rays are the most energetic EM waves, they have frequencies higher than 10^{20} Hz, estimate energy of a γ-photon:

$$E = hf > 6.63 \times 10^{-34} \times 10^{20} \approx 10^{-13} \text{ J}$$

Einstein relation: comments

1. $E \propto f \Rightarrow$ higher/lower frequency, higher/lower energy
2. relate energy to wavelength: $c = \lambda f \Rightarrow \boxed{E = \frac{hc}{\lambda}}$

   $E \propto \frac{1}{\lambda} \Rightarrow$ longer/shorter wavelength, lower/higher energy
3. Einstein relation tells connection between a particle property (energy E) and a wave property (frequency f, or wavelength λ)

explanation of photoelectric effect

electrons are trapped in an energy well (electrostatic attraction between positive metal ions and negative electrons)

an electron requires a minimum energy, called work function Φ, to escape a metal's surface

an electron may capture a photon and absorb all its energy
if this energy is greater than Φ, the electron can break free from metal

photoelectric equation

Einstein's photoelectric equation:

$$\boxed{hf = \Phi + E_{k,\mathsf{max}} = \Phi + \frac{1}{2}mv^2_{\mathsf{max}}}$$

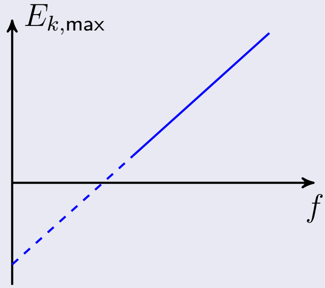
at critical condition, K.E. of emitted electron is zero

threshold frequency f_0 is given by: $\boxed{hf_0 = \Phi}$

further comments on photon theory

1. single photon interacts with single electron, one-to-one interaction event
2. photon no longer exists after being absorbed by electron
3. greater intensity means more photons per unit time, so more electrons released
4. greater frequency means more energetic photons, so more energetic electrons
5. below threshold f_0, electron cannot obtain enough energy to overcome Φ and hence cannot escape

graphical interpretation of photoelectric equation



rearrange equation:

$$E_{k,\text{max}} = hf - \Phi$$

plot $E_{k,\text{max}}$ against f, then

$$\text{gradient} = h$$

$$x\text{-intercept} = \frac{\Phi}{h} = f_0$$

$$y\text{-intercept} = -\Phi$$

if measurements for a set of $(f, E_{k,\text{max}})$ are taken, plot a best-fit line, then Planck constant h, threshold frequency f_0, work function Φ can all be computed

electronvolt

define a more convenient energy unit electronvolt (eV):

energy transferred when an electron travels through a potential difference of one volt

$e = 1.60 \times 10^{-19}$ C $\Rightarrow$ $\boxed{1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}}$

Example

Find the longest wavelength of light waves that release electrons from gold. (Work function of gold $\Phi = 4.9$ eV)

$$f_0 = \frac{\Phi}{h} = \frac{4.9 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}} \approx 1.18 \times 10^{18} \text{ Hz}$$

$$\lambda_0 = \frac{c}{f_0} = \frac{3.00 \times 10^{8}}{1.18 \times 10^{15}} \approx 2.5 \times 10^{-7} \text{ m} \text{ (ultraviolet light)}$$

worked example

Why our demonstration does not work?

Given that the work function energy for zinc is 4.3 eV, and the wavelength of the ultraviolet torch emits is 365 nm. Explain why this wavelength does not cast any effect on the charged metal leaf of the electroscope.

photon energy: $E = hf = \dfrac{hc}{\lambda}$

$$\Rightarrow E = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{3.65 \times 10^{-7}} \approx 5.45 \times 10^{-19} \text{ J}$$

work function: $\Phi = 4.3 \times 1.6 \times 10^{-19} \approx 6.88 \times 10^{-19}$ J

$E < \Phi$, so not enough energy to release electrons

summary of photoelectric effect

basic principle: electron absorbs photon, acquires energy, overcomes work function, and escapes from metal surface

photoelectric equation: $hf = \Phi + \frac{1}{2}mv^2_{\text{max}}$

phenomenon	explanation
immediate effect	one-to-one collision event
$f \nearrow$, more energetic electrons	photon energy $E \propto f$
threshold frequency	$E \propto f$ and $E > \Phi$
intensity $\nearrow$, more electrons	photon number $\propto$ intensity

dual nature of light

the dual nature of light

- travels through space as a wave (electromagnetic wave) $\rightarrow$ interference, diffraction
- interacts with matter as a particle (photon) $\rightarrow$ photoelectric effect, emission/absorption spectrum of element, Compton scattering

wave-particle duality: light shows both wave-like and particle-like behaviours, it is both a wave and a particle

matter waves

Louis de Broglie suggested in 1924 that all matter should also have a dual nature

de Broglie was awarded the 1929 Nobel Physics Prize "*for his discovery of the wave nature of electrons*"

wave-like property can be represented by wavelength λ

de Broglie wavelength: $\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$

where $p = mv$ is momentum of the particle

de Broglie wavelength

Example

estimate de Broglie wavelength of Colin walking at 2 m·s^{-1} and electrons travelling at 10^6 m·s^{-1}

$$\lambda_{\text{Colin}} = \frac{h}{m_{\text{Colin}} v} = \frac{6.63 \times 10^{-34}}{60 \times 2} \approx 5.5 \times 10^{-36} \text{ m}$$

$$\lambda_e = \frac{h}{m_{\text{e}} v} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^6} \approx 7.3 \times 10^{-10} \text{ m}$$

λ_{Colin} is extremely small, so I don't look like a wave at all!
but fast-moving electrons can be diffracted by solids (atomic spacing $\sim 10^{-10}$ m)

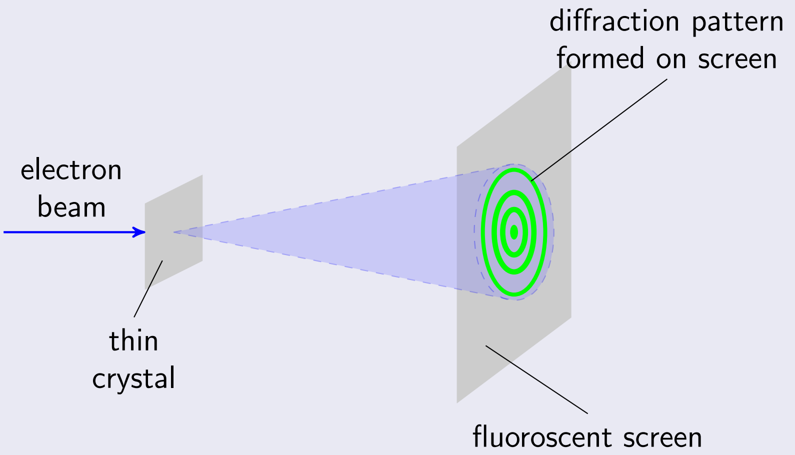
electron diffraction

wave property of electron was confirmed by Clinton Davisson and George Thomson in 1927

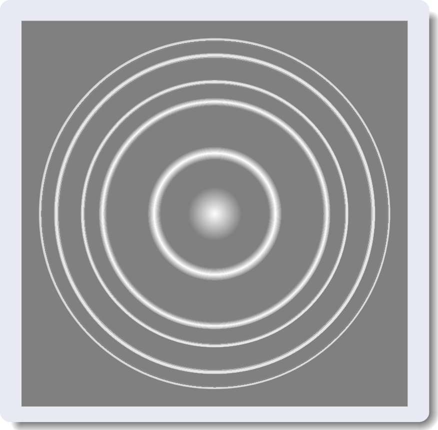
they showed experimentally electrons were diffracted by metal (nickel) crystals

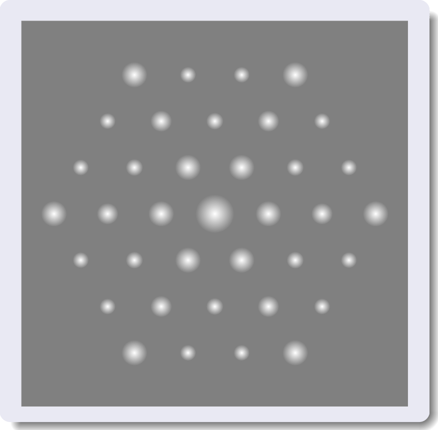
1937 Nobel Physics Prize was awarded jointly to Clinton Davisson and George Thomson "*for their experimental discovery of the diffraction of electrons by crystals*"

electron diffraction experiment



electron diffraction patterns





applications of electron diffraction

can investigate structure of matter using electron diffraction

electrons of various wavelengths are used to explore arrangements of atoms, structures of complex molecules, structure of atomic nuclei, etc.

electron microscope, using electrons beams with wavelengths 10^5 shorter than visible light, better than 50 pm resolution, magnifications up to 10^7x

momentum of light

de Broglie relation also applies to photons, this predicts a photon has momentum $\boxed{p = \frac{h}{\lambda}}$

compare with Einstein relation $E = hf = \frac{hc}{\lambda}$, photon has energy spectrum $\boxed{E = pc}$

notice that this is a relativistic momentum, as photons move at speed of light, definition for classical momentum $p = mv$ does not apply for photons

example question: radiation pressure

radiation pressure on a surface

A laser of power $P = 5.0$ mW incident on a spot of $A = 2.0$ mm^2, what is the pressure caused by the beam?

number of photons incident in Δt is:

$$\Delta N = \frac{\text{total energy of beam}}{\text{energy of each photon}} = \frac{P\Delta t}{hf}$$

each photon exerts a force:

$$F_0 = \frac{\text{change in momentum}}{\text{time taken}} = \frac{\Delta p}{\Delta t} = \frac{h}{\lambda \Delta t}$$

pressure caused by laser beam:

$$p = \frac{F}{A} = \frac{\Delta N \times F_0}{A} = \frac{P\Delta t}{hf} \times \frac{h}{\lambda \Delta t} \times \frac{1}{A} \Rightarrow p = \frac{P}{cA}$$

$$p = \frac{5.0 \times 10^{-3}}{3.0 \times 10^{8} \times 2.0 \times 10^{-6}} = 8.3 \times 10^{-6} \text{ Pa}$$

wave-particle duality

electromagnetic radiation

- particle-like behaviour: photoelectric effect, radiation pressure
- wave-like behaviour: interference, diffraction, Doppler effect, reflection & refraction

matter particles

- particle-like behaviour: deflection in electric/magnetic fields, decay, scattering
- wave-like behaviour: electron diffraction

wave-particle duality and beyond

everything has a universal dual nature

light/electron/atom/bird/earth ... both a wave and a particle

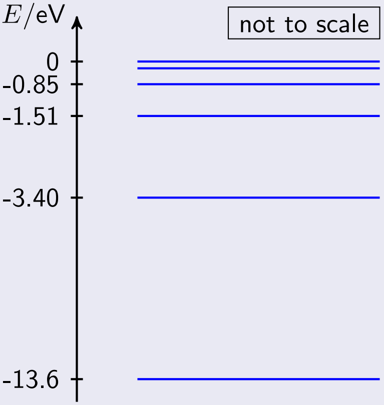
$\Rightarrow$ wave-particle duality

wave-particle duality addresses breakdown of classical concepts like 'particle' or 'wave'

to fully describe the behaviour of microscopic objects, need rewrite theory of classical mechanics $\longrightarrow$ quantum mechanics

"The nature is not only queerer than we suppose, but queerer than we can suppose." — J.B.S. Haldane, Possible Worlds

quantised energy states



energy levels of a hydrogen atom

electrons in an atom only take certain orbits, corresponding to certain fixed values of energy

energy of electrons in an atom is discrete/quantised, called energy levels

comments on energy level

1. energy levels are always negative
   external energy needed to remove electrons from an atom
2. an electron with zero energy is a free electron
3. meaning of quantisation
   an electron can have only one of these values of energy
   cannot have an energy between energy levels
   e.g., only E_1, E_2, E_3, but no intermediate values

Bohr's theory of hydrogen atom

theoretical explanation for energy levels was developed in 1913 by Danish physicist Niels Bohr in his theory of hydrogen atom

Neils Bohr received the Nobel Physics Prize in 1922 "*for his services in the investigation of the structure of atoms and of the radiation emanating from them*"

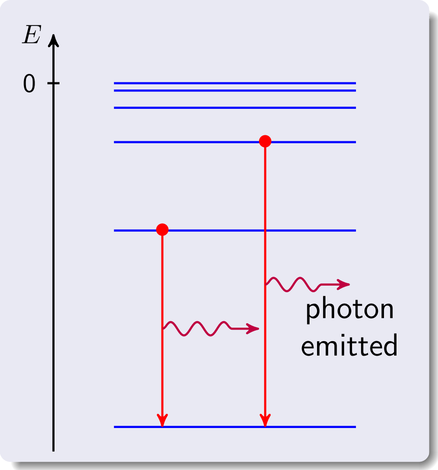
electron transitions

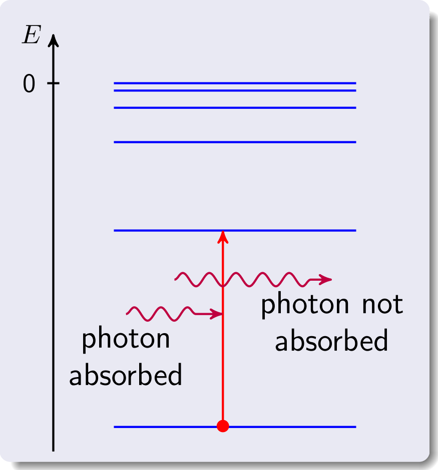
an electron can jump between energy levels $\longrightarrow$ transitions

transition to lower level $\rightarrow$ lose energy $\rightarrow$ emit photon
transition to higher level $\rightarrow$ require energy $\rightarrow$ absorb photon with the correct energy

quantised energy levels $\Rightarrow$ only certain wavelengths permitted
this gives rise to discrete emission/absorption line spectra

emission/absorption of photons





discrete spectrum: emission spectrum

hot gas of an element gives a unique collection of sharp and bright lines seen in the spectrum $\longrightarrow$ emission line spectrum

emission spectrum is a discrete spectrum

emission spectrum can be used to identify elements

explanation of emission spectrum

electrons excited to higher energy states by fast-moving free electrons (accelerated by high voltage)

when excited electrons return to ground states, they emit photons of certain frequencies

each frequency corresponds to a line in emission spectrum

example question: transition of electrons

transition of electrons

An electron falls from the second level (-3.40 eV) to lowest energy level (ground state) (-13.6 eV) of a hydrogen atom. Find the wavelength of the light emitted.

$$hf = \Delta E \Rightarrow f = \frac{E_2 - E_1}{h}$$

$$f = \frac{(-3.40 \text{ eV}) - (-13.6 \text{ eV})}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \approx 2.46 \times 10^{15} \text{ Hz}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{2.46 \times 10^{15}} \approx 1.22 \times 10^{-7} \text{ m (ultraviolet band)}$$

example question: hydrogen spectrum

Hydrogen spectrum

Neutral hydrogen atoms are excited to the third level. The atoms then de-excite, emitting light until settling to the ground state.

How many emission lines are observed?

Which transition gives out emission line with shortest wavelength?

possible transitions: $E_3 \to E_2$, $E_2 \to E_1$, $E_3 \to E_1$

so a total of 3 emission lines

$\frac{hc}{\lambda} = \Delta E$, smallest λ so greatest ΔE, this corresponds to the transition $E_3 \to E_1$

fluorescent tubes ($\star$)

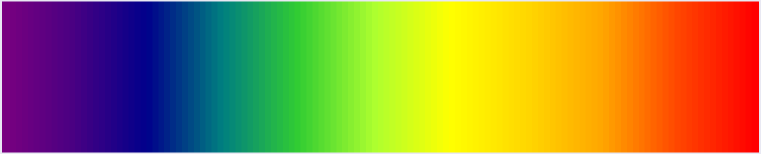
fluorescent tubes contain mercury vapour, across which a high voltage is applied, electrons are excited and then return to ground states, emitting photons in UV range

a fluorescent material (e.g., a phosphorus coating) on inside of tube absorbs these photons, exciting its electrons to much higher states, these electrons then cascade down the energy levels, emitting many photons in the form of visible light

continuous spectrum

splitting light (with a prism, diffraction grating, etc.) into its different wavelengths, this process is called dispersion of light, this gives a spectrum

white light (eg. sunlight) consists of a range of wavelengths or frequencies $\longrightarrow$ giving a continuous spectrum

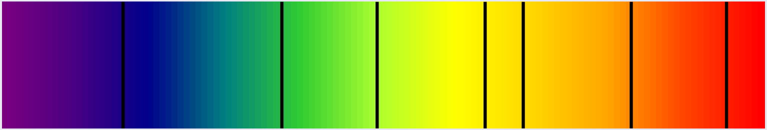


discrete spectrum: absorption spectrum

pass white light through a cool gas, certain wavelengths are missing, dark lines appear in background of continuous spectrum $\longrightarrow$ absorption line spectrum

absorption spectrum is a discrete spectrum

absorption spectrum is used to study components of stars



explanation of absorption spectrum

at low temperatures, most electrons stay in ground states

pass a light through cold gas, photons of correct wavelength are absorbed by electrons to excite them to higher energy levels

these wavelengths are then missing, corresponding to lines in absorption spectrum

transition and photon emission/absorption

energy of photon to be emitted/absorbed must match the gap between two energy levels

energy conserved during electron and photon interaction

$\boxed{hf = \Delta E}$ (for both emission/absorption processes)

example question: counting dark lines

A white light of wavelengths $420 \sim 740$ nm passes through a cool gas cloud. Electron energy levels of gas atoms are: $E_1 = -13.6$ eV, $E_2 = -3.41$ eV, $E_3 = -1.50$ eV, and $E_4 = -0.85$ eV. How many dark lines can be seen in the emergent spectrum?

use $E = \dfrac{hc}{\lambda}$ to find range of photon energies: $1.68 \sim 2.96$ eV

possible absorptions: $E_2 \rightarrow E_3\,(1.91$ eV$)$, $E_2 \rightarrow E_4\,(2.56$ eV$)$
so two dark lines in the absorption spectrum

summary of energy levels

electrons can only take discrete energy levels

when electrons make transitions from one level to another, energy is released or absorbed

this is related to emission or absorption of photon with the correct frequency

quantised energy levels for electrons, so only photons or certain wavelengths are emitted or absorbed

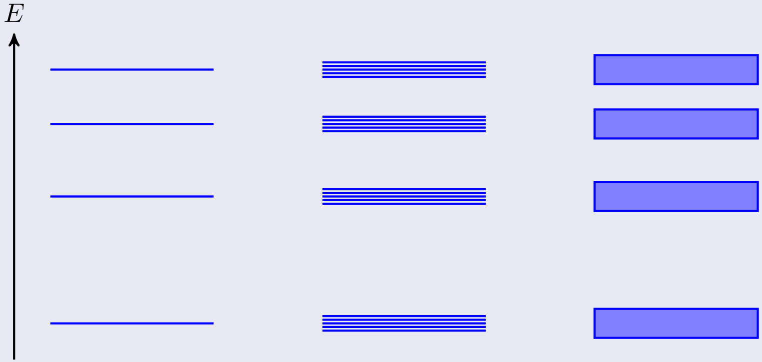
energy bands

in an isolated atom, electrons have discrete energy levels

in solids, interaction between neighbouring atoms causes change in configuration of energies
original energy levels split into a band with many sub-levels

number of sub-levels in each band equals number of atoms in solid
very large number of atoms present, so sub-levels (seem to) form continuous energy bands

energy bands



dicrete energy levels in isolated atom

split energy sub-levels due to interactions

energy bands formed in many-particle system

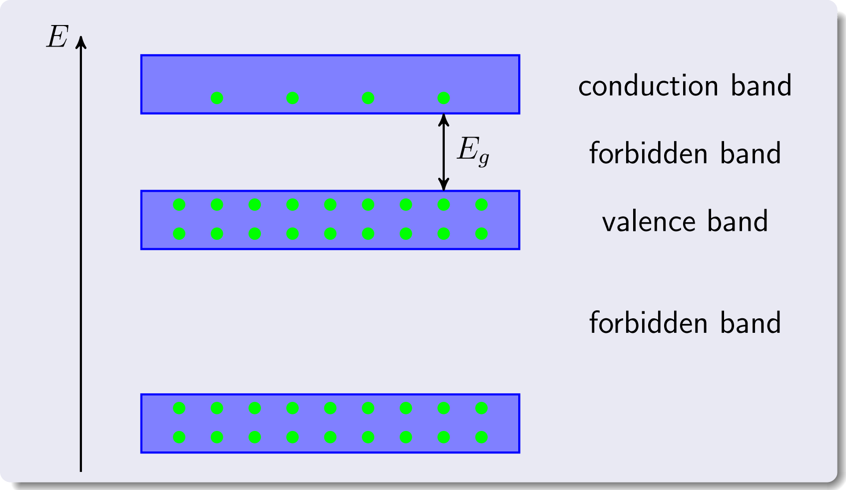
band structure

forbidden bands represent energies that electrons are not allowed to take

electrons fill up from lowest energy bands available
highest fully filled band is called valence band
next partially filled or empty band is called conduction band

energy separation between valence band and conduction band defines energy gap of material

band structure



band theory & electrical conductivity

electrical conductivity of material can be explained by its band structure

energy bands up to valence band in a solid are fully filled, all states available are occupied, these electrons cannot move freely to form electric currents
so the number of free electrons available in conduction band is the key to electrical conductivity

insulators

conduction band of an insulator is empty

valence band is fully filled

so no movement of electron is possible

insulators also have large energy gaps E_g, so thermal or photo-excitation cannot easily make valence electrons jump into conduction band

so insulators are poor conductors of electric currents

semi-conductors

band structure of semi-conductors is similar to insulators: an empty conduction band and a fully-filled valence band
but semi-conductors have much narrower gaps

as temperature rises, valence electrons gain thermal energy, these excited electrons cross the energy gap and enter conduction band

charge carriers in semi-conductors

electrons entering conduction band become free electrons that can flow through material

at the same time, holes[a] are formed in valence band

[a] A hole has a positive charge, since an electron is missing from its site. Neighbouring electrons can move to fill the hole and leave a new hole, but this can be thought of as the hole moves about.

there are negative (electrons) as well as positive (holes) charge carriers in semi-conductors

more charge carriers available, this contributes to conductivity

charge carriers in semi-conductors

lattice vibration would also increase at higher temperature
this causes greater scattering of charge carriers

effect of more charge carriers is far greater than lattice vibration, so resistance decreases with temperature

now you understand why thermistors have lower resistance at higher temperature

example question: energy gap calculations

good conductor or bad conductor?

Diamond and silicon, have energy gap $E_g \approx 6.0$ eV and 1.1 eV respectively, argue whether they conducting when exposed to red light of 600 nm.

energy of radiation:

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{600 \times 10^{-9}} \approx 3.32 \times 10^{-19} \text{ J} \approx 2.1 \text{ eV}$$

sufficient to make valence electrons in silicon to cross energy gap, so silicon becomes conducting

but not enough to excite valence electrons in diamond, so diamond remains insulator and transparent to red light

metallic conductors

conduction band of typical metal overlaps with valence band
no band gap means there are vacant states for conduction electrons to occupy at ease
this means conduction electrons are free to move around to form currents, so metals are good conductors

as temperature rises, almost no change in number of conduction electrons, but lattice vibration increases, electrons are more likely to be scattered, so resistance would increase

comments on conductors

- metals are opaque to visible light or other low-frequency radiation

  width of conduction band for typical metal is about $2 \sim 3$ eV

  low-energy photons can be absorbed by conduction electrons

- metals are transparent to high-frequency radiation such as X-rays

  if X-ray photons are absorbed, electrons would enter forbidden band

  but this is not allowed, so high-frequency photons penetrate through

- as conduction electrons flow, they dissipate energy as heat

  they give off kinetic energy as they get scattered by metal ions

  thermal vibration of metal lattice increases, causing heating effects

summary of band theory

for thermistor/LDR, VB is fully filled, CB is empty
as T/light intensity $\nearrow$, VB electrons absorb energy
they cross band gap and enter CB
free electrons become available in CB, holes are formed in VB
more charge carriers available, so resistance decreases

for metallic conductors, CB and VB overlap (no band gap)
as T $\nearrow$, no change in number of free electrons
but greater thermal vibration
so resistance of metal increases