Finite element analysis of composite structures

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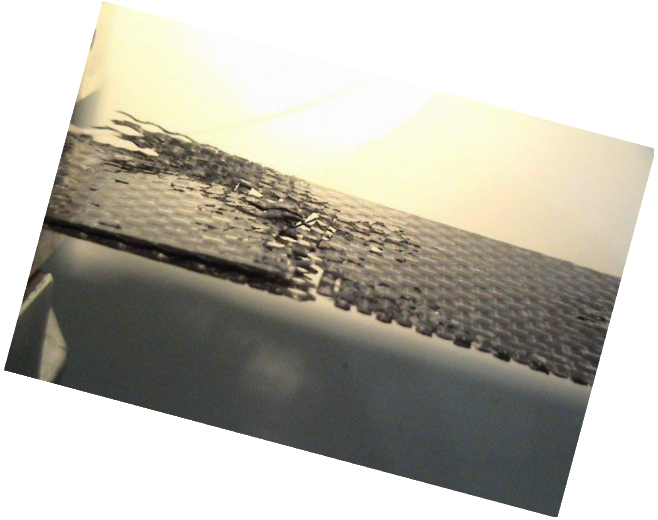
Overview – FEA of composite structures

- Introduction
  - Typical composite applications
- Composite materials
  - Fibre and matrix properties
  - Fabrics and preforms
- Unidirectional composite
  - Material properties
- Layered structures
  - ABD matrix and its implications
- FEA of composite structures
  - Elements for FEA of composites
  - Stress and strength
- Examples



Introduction – FEA of composite structures

- Finite element analysis of composite structures
  - The principle of FEA same as for the isotropic materials from the previous courses

$$\mathrm{K}.u = f$$

  - K global stiffness matrix
  - *u* global vector of nodal displacements
  - *f* global vector of external equivalent nodal forces

solution: $u = K^{-1}.f$

  - For composite structures more challenging in pre-processing of models and post-processing of results
    - Due to orthotropic behaviour of material and other important parameters
  - In this lecture, the basic approaches for modelling of long fibre reinforced plastics are discussed

Introduction – FEA of composite structures

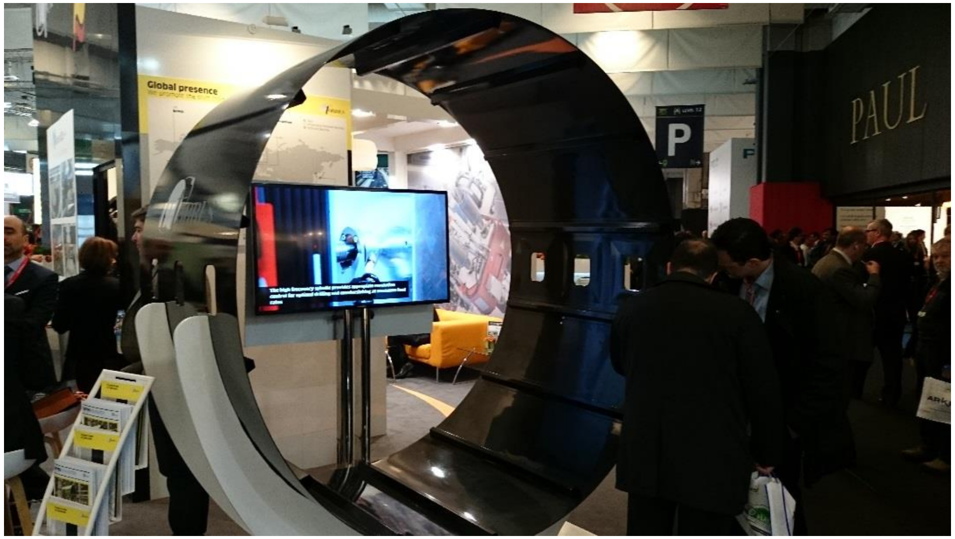
- Important to know
  - What are the demanded results of the simulation?
    (stress, displacement, natural frequencies, temperature distribution, crash behaviour, …)
  - What is the demanded precision of results?
  - What manufacturing technology and preforms are used for the structure?
    - fabrics, prepregs, fibre tows
    - unidirectional versus multidirectional preform
    - abilities of manufacturing technology

- Important decisions
  - Elements type selection and geometry simplifications
  - Modelling of composite structure
    - full composite lay-up
    - ABD matrix
    - properties homogenization

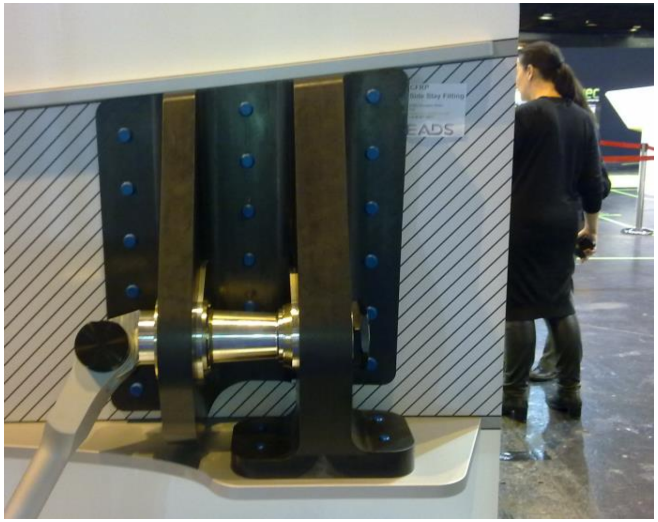
Introduction – Composite structures - Aerospace

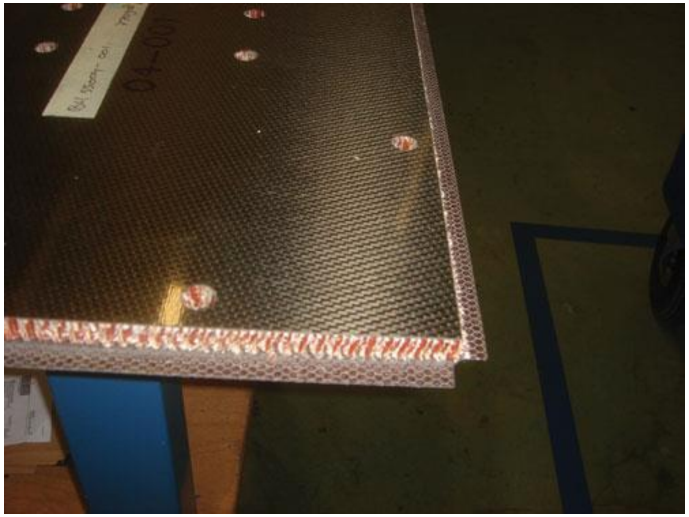
- Airbus 350XBW, Premium AEROTEC

Introduction – Composite structures - Aerospace

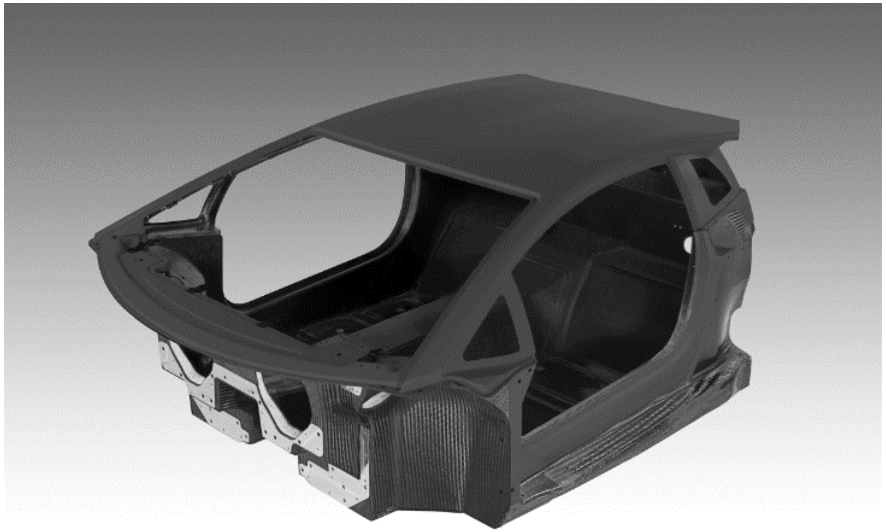


Introduction – Composite structures - Aerospace





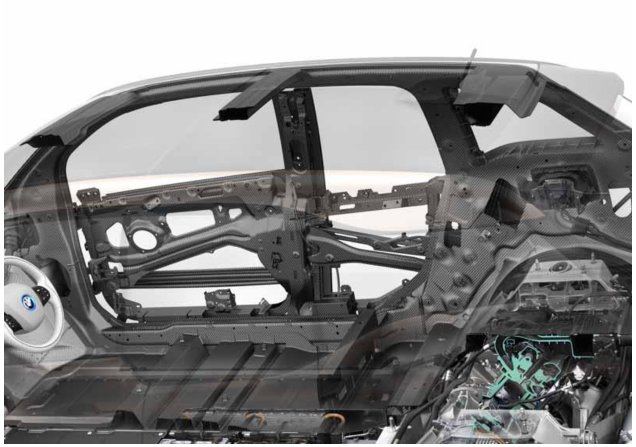
Introduction – Composite structures - Automotive



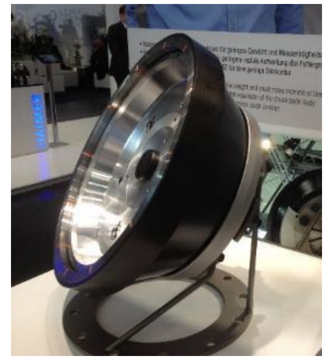
Introduction – Composite structures - Automotive

Introduction – Composite structures - Aerospace

- BMW I3
  - CFRP life module
  - weight reduction
  - since 2013 on sale



Introduction – Composite structures - Industry



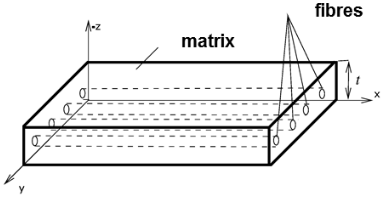
Introduction – Composite structures

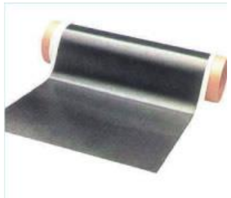
Introduction

- Short conclusions in terms of composite structures
  - Usually thin components (thickness is significantly smaller than other 2 dimensions)
    - Suitable for shell elements, beam elements
    - Options for solid modelling limited
  - Usually composite lay-up with layers with multiangle orientations, structures with only 1 orientation of fibres are rare
  - Various semi-finished products used in the structures
    - Fabrics, prepregs, rovings
    - Different manufacturing technologies, different fibre volume fraction in the composite layer
  - Various materials used in applications
    - Fibres
    - Matrices
  - All of the aforementioned influence the behaviour of the component and thereby the demands for its modelling

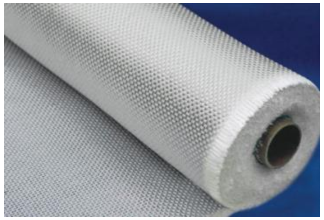
Composite materials

- Demonstration – layer of composite material





- Properties of layer is determined by
  - type of fibre
    - *carbon, glass, boron, aramid*
  - type of matrix
    - *thermosets - epoxy,…*
    - *thermoplastics – PA12, PEEK, PPS,...*
  - type of semi-finished product
    - *unidirectional*
    - *multidirectional*
  - fibre volume fraction in the layer
    - manufacturing technology



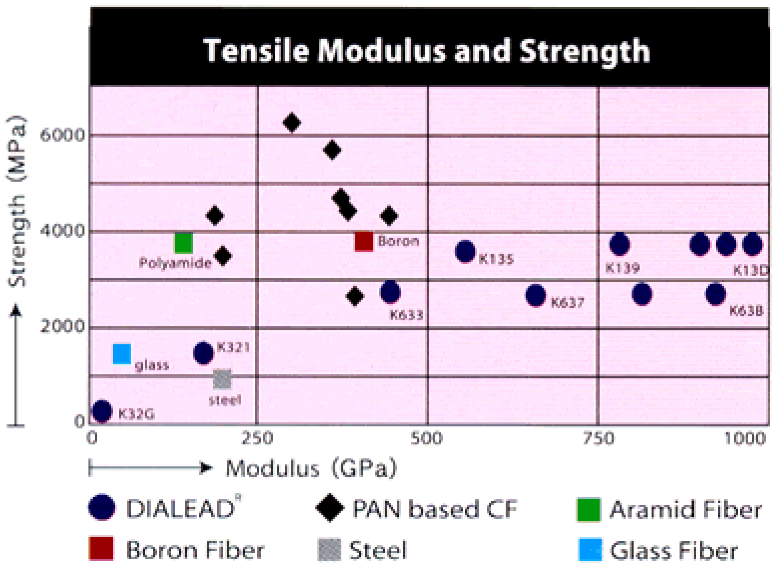
Composite materials - Fibres

- Fibres
  - carry the load; significantly defines the stiffness
- Overview of nominal properties of selected types of fibres
  - E_L – Youngs' modulus in direction of fibre
  - E_T – Youngs' modulus in direction perpendicular to fibre
  - G_{LT} – shear modulus of fibre
  - σ_{Lf} – tensile strength of fibre
  - α_L – thermal expansion coefficient in direction of fibre
  - λ_L – thermal conductivity in direction of fibre
- Glass – isotropic fibre, Carbon – strongly anisotropic

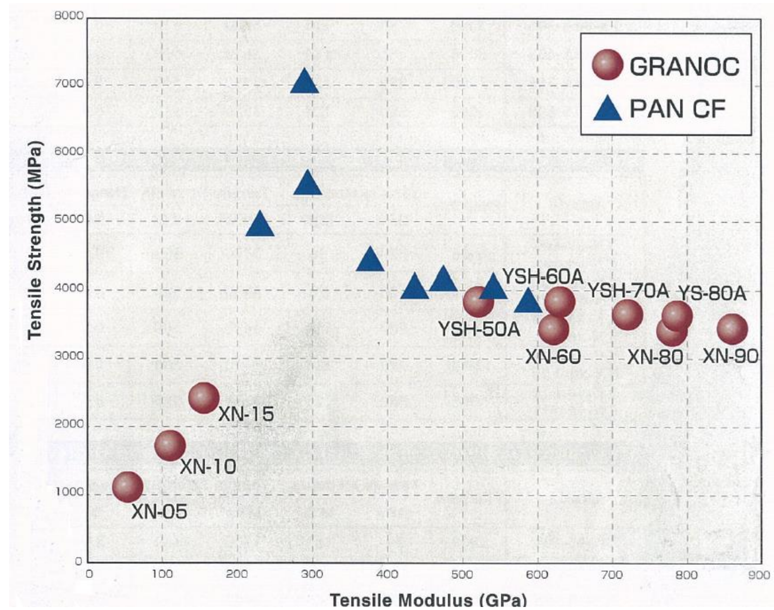
	ρ_L [kg.m^{-3}]	E_L [GPa]	E_T [GPa]	G_f [GPa]	σ_{Lf} [MPa]	α_L [K^{-1}]	λ_{Lf} [W.m^{-1}.K^{-1}]
High-strength PAN carbon	1800	230	*15*	*50*	4900	-0,38e-6	10
Ultra-high modulus PITCH carbon	2170	780	*5*	*20*	3200	-1,5e-6	320
E-glass	2580	72	72	*30*	3400	5,4e-6	1,35
S-glass	2460	87	87	*38*	4900	1,6e-6	1,45
Aramid	1440	124	*5*	*12*	2800	-2,4e-6	0,04

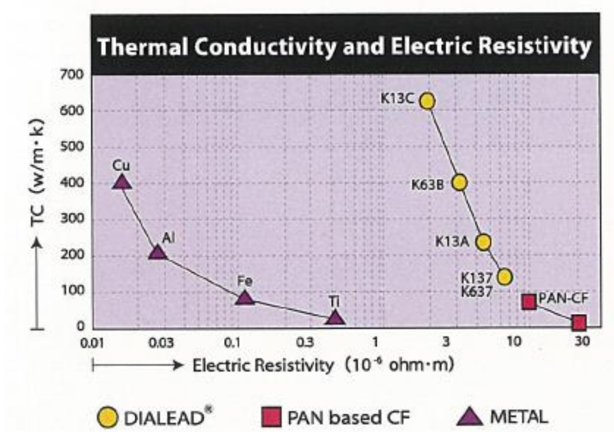
Composite materials - Fibres

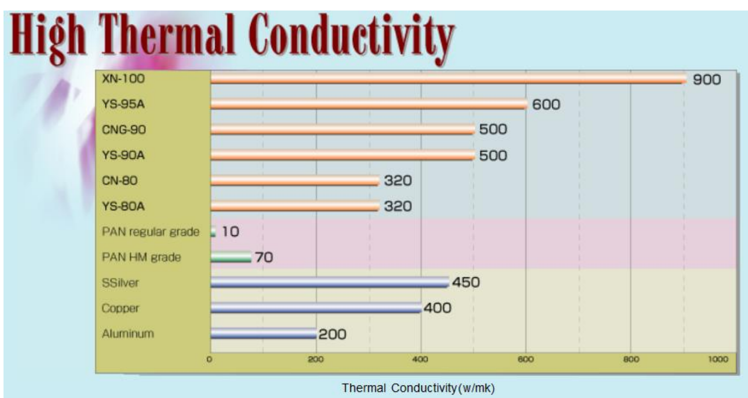
DIALEAD - Mitsubishi Plastics



Nippon Graphite Fiber Corporation







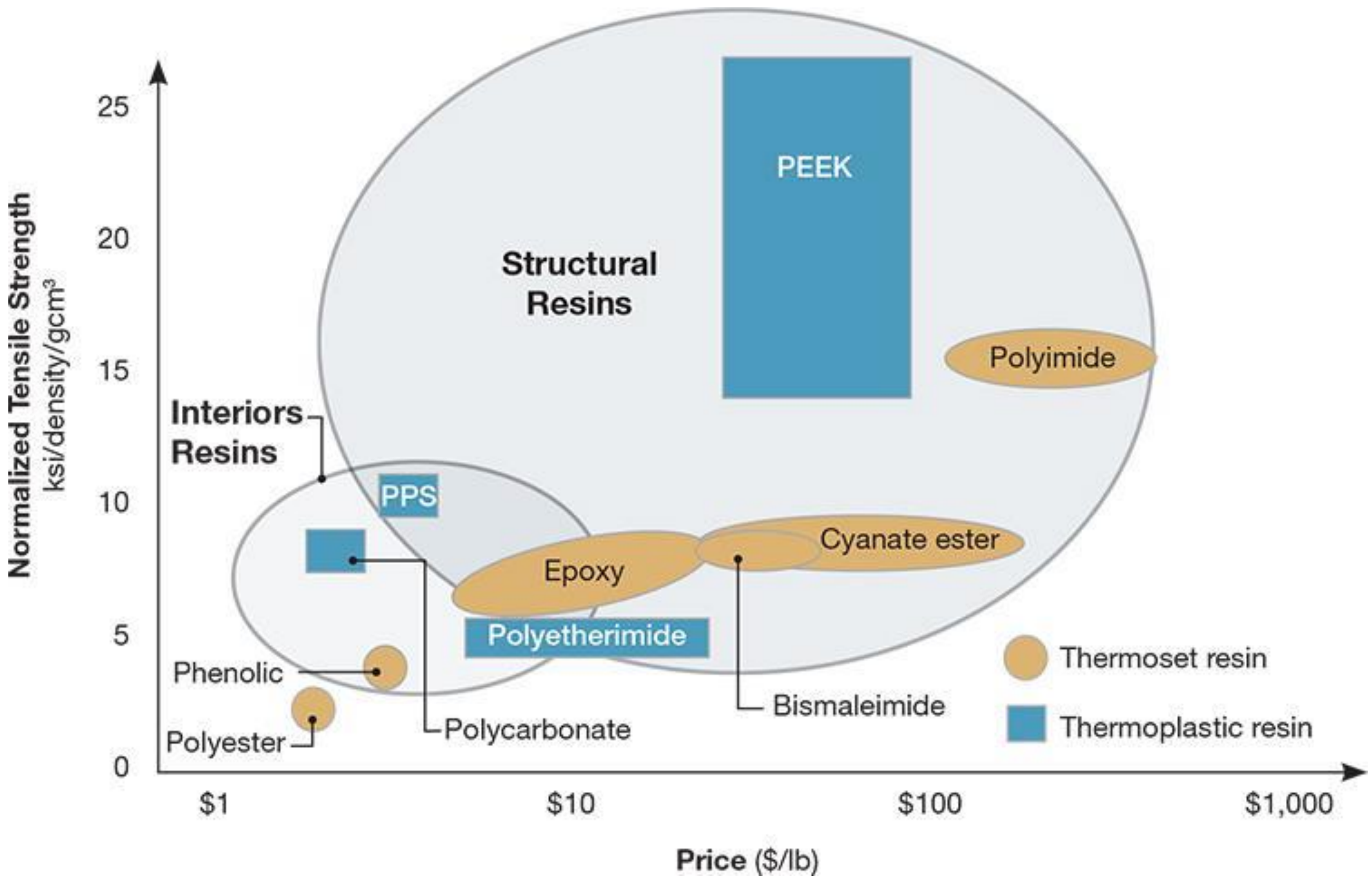
Composite materials - Matrices

- Matrix
  - affects strength, fracture toughness
  - affects other properties (flammability, conductivity, fatigue, biocompactibility, …)
  - determines/restricts the manufacturing technologies

- **Thermosets**
  - **non-repeatable manufacturing process**
    - after curing no reshaping (non-destructively)
  - **longer time of curing (hours… *minutes*)**
  - brittle materials
  - …

- **Thermoplastics**
  - **repeatable manufacturing process**
    - after heating – matrix softening – reshaping
  - **short time of processing (minutes)**
  - good fracture toughness
  - …

Composite materials - Matrices



Source: RED, Chris. The Outlook for Thermoplastics in Aerospace Composites, 2014-2023. In High-Performance Composites. Vol. 22, No. 5, 2014.

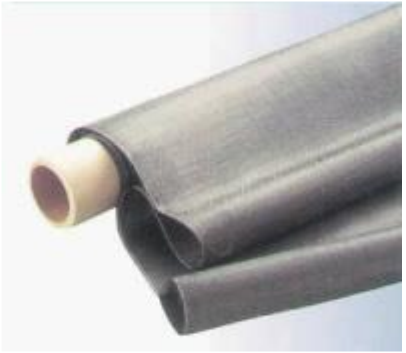
Composite materials - Matrices

Matrix	Density [kg.m^{-3}]	E [MPa]	α [K^{-1}]	λ [W/m/K]	Glass transition temp. [°C]	Melting temp. [°C]
Epoxy	1150	2600÷5000	60e-6	0,2÷0,5	50÷200	x
Non-saturated polyesters	1170÷1260	14000÷20000	20÷40e-6	0,3÷0,7	60÷170	x
Phenolic resins	1400÷1800	5600÷12000	15÷50e-6	0,4÷0,7	70÷120	x
PP	900	1300-1800	130÷180e-6	0,17÷0,25	-20÷20	160÷165
PA6	1150	2800	80÷90e-6	0,22÷0,3	45÷80	225÷235
PA12	1004	1400	120÷140e-6	0,22÷0,24	40÷50	170÷180
PPS	1350	3700	50÷70e-6	x	85÷100	275÷290
PEEK	1300	3700	50÷70e-6	0,25	145÷155	335÷345
PEI	1270	3000	50e-6	0,22	215÷230	x

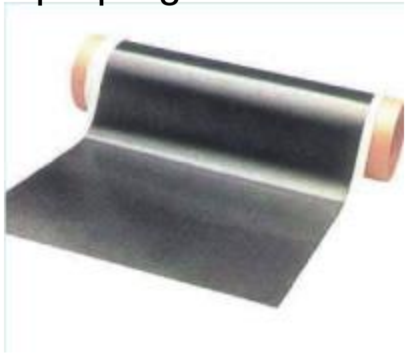
Composite materials - semi-finished products

- Possible semi-finished products for composite applications
  - fabrics
  - prepregs, UD tapes
  - rovings / fibre tows
  - chopped fibres
- Properties of semi-finished product influences the behavior of the unit and component (stiffness, strength)
  - fibre orientation – uni or bi-directional
  - amount of fibres
- Type of semi-finished product affects the modelling approach

fabrics



prepregs



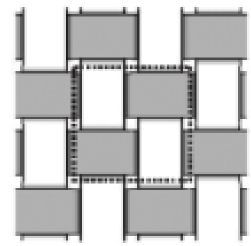
rovings

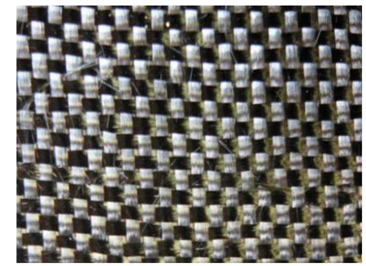
chopped fibres

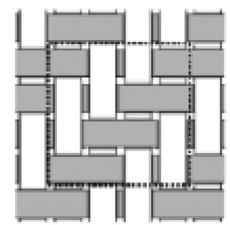
Source: http://www31.ocn.ne.jp/~ngf/english/product/index.htm#p2

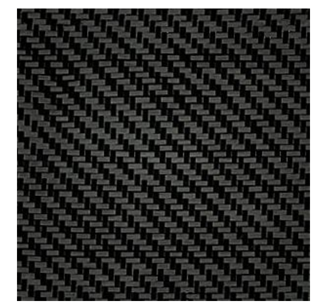
Composite materials - semi-finished products

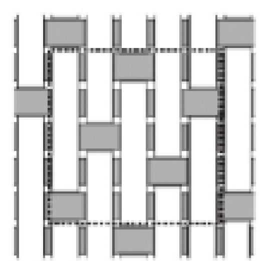
- Fabrics
  - plain
    - worse drapability
    - good strength, resistance against shift of fibres
  - twill
    - average drapability
  - satin
    - good drapability
    - small resistance against shift of fibres

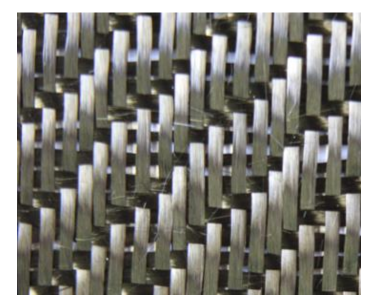

plain




2/2 twill



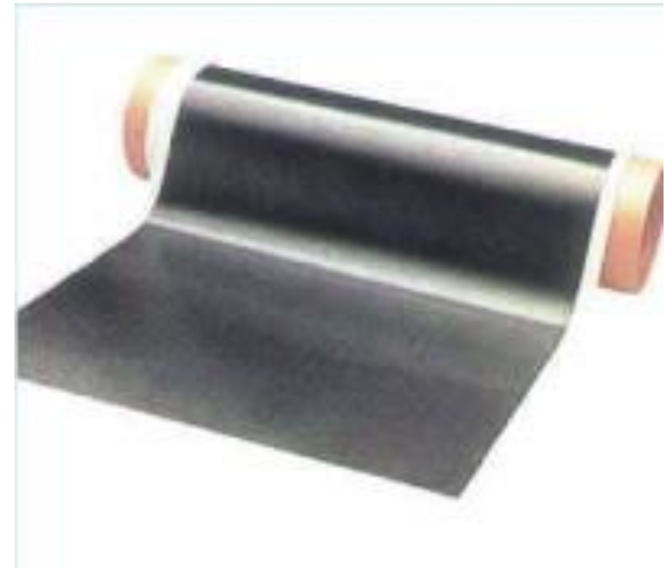

satin

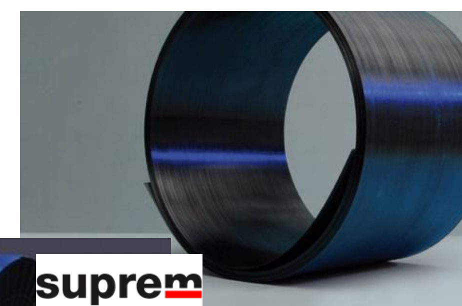


Composite materials - semi-finished products

- Prepregs
  - thermosets
    - fabric or uni-directional and semi-cured matrix
  - thermoplastics
    - fabric or uni-directional and thermoplastic matrix
- Storage
  - thermosets
    - must be stored at approx. -18°C, limited lifetime
  - thermoplastics
    - can be stored at room temperature, without lifetime restrictions

- One of the highest-quality semi-finished product



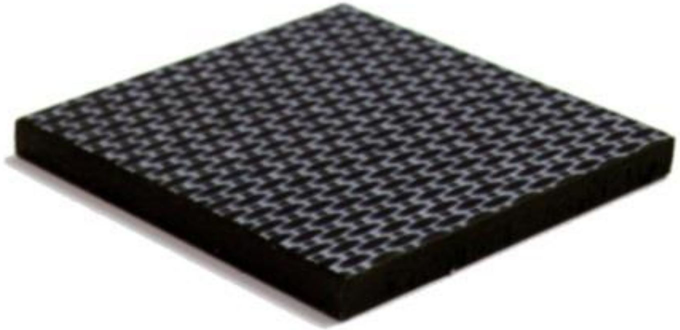


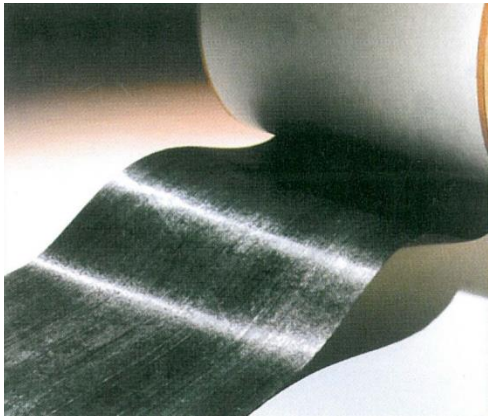
Composite materials - semi-finished products

- Rovings
  - fibre tows
  - notation 1k, 3k, 6k, 12k, 24k, 48k gives number of fibres in the tow (1k ~ 1000 fibres)
  - for filament winding, fibre placement, manufacturing of prepregs and fabricss

Composite materials

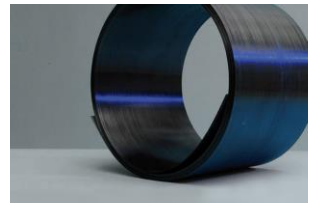
- Short overview
  - Internal structure of composite layer determines mechanical properties
    - stiffness
    - strength
    - other…
  - From the FEA point of view
    - Properties of layer described by material, thickness and orientation
    - However, care must be taken when simplifying the semi-finished products like bi-directional fabrics into the layer properties
    - Basic unit for simulations – Uni-directional layer of composite

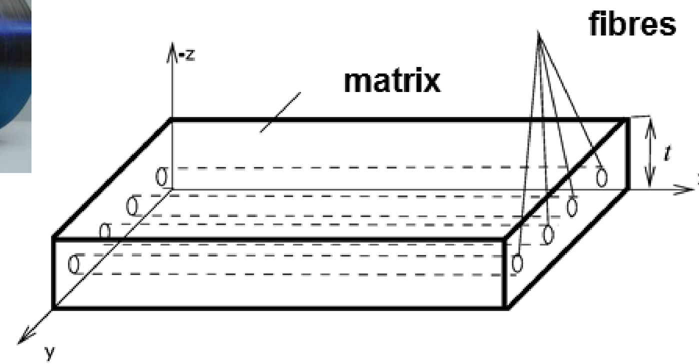




Unidirectional layer of composite

- Basic computational element
- Properties determined by
  - type of fibre
  - type of matrix
  - fibre volume fraction in the layer
  - thickness of the layer





isotropic material

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E & -\upsilon/E & -\upsilon/E & 0 & 0 & 0 \\ -\upsilon/E & 1/E & -\upsilon/E & 0 & 0 & 0 \\ -\upsilon/E & -\upsilon/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} . \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}$$

Parameters: E, ν

$$G = \frac{E}{2.(1+\upsilon)}$$

orthotropic material

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\upsilon_{21}/E_2 & -\upsilon_{31}/E_3 & 0 & 0 & 0 \\ -\upsilon_{12}/E_1 & 1/E_2 & -\upsilon_{32}/E_3 & 0 & 0 & 0 \\ -\upsilon_{13}/E_1 & -\upsilon_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} . \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}$$

Parameters: E_x, E_y, E_z, G_{xy}, G_{xz}, G_{yz}, ν_{xy}, ν_{xz}, ν_{yz}

$$\upsilon_{ij} / E_i = \upsilon_{ji} / E_j$$

Unidirectional layer of composite

- Material properties must fulfil stability criterion

isotropic material

$$\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{Bmatrix}=\begin{bmatrix}1/E & -\upsilon/E & -\upsilon/E & 0 & 0 & 0\\ -\upsilon/E & 1/E & -\upsilon/E & 0 & 0 & 0\\ -\upsilon/E & -\upsilon/E & 1/E & 0 & 0 & 0\\ 0 & 0 & 0 & 1/G & 0 & 0\\ 0 & 0 & 0 & 0 & 1/G & 0\\ 0 & 0 & 0 & 0 & 0 & 1/G\end{bmatrix}.\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{Bmatrix}$$

orthotropic material

$$\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{Bmatrix}=\begin{bmatrix}1/E_1 & -\upsilon_{21}/E_2 & -\upsilon_{31}/E_3 & 0 & 0 & 0\\ -\upsilon_{12}/E_1 & 1/E_2 & -\upsilon_{32}/E_3 & 0 & 0 & 0\\ -\upsilon_{13}/E_1 & -\upsilon_{23}/E_2 & 1/E_3 & 0 & 0 & 0\\ 0 & 0 & 0 & 1/G_{12} & 0 & 0\\ 0 & 0 & 0 & 0 & 1/G_{13} & 0\\ 0 & 0 & 0 & 0 & 0 & 1/G_{23}\end{bmatrix}.\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{Bmatrix}$$

Parameters: E, ν

$$G=\frac{E}{2.(1+\upsilon)}$$

Parameters: E_x, E_y, E_z, G_{xy}, G_{xz}, G_{yz}, ν_{xy}, ν_{xz}, ν_{yz}

$$\upsilon_{ij}/E_i=\upsilon_{ji}/E_j$$

Conditions of stability

E>0, G>0

$$-1<\upsilon<0{,}5$$

$E_i>0$, $G_{ij}>0$ i,j=x,y,z

$$|\upsilon_{ij}|<\sqrt{E_i/E_j}$$

$$1-\upsilon_{xy}\upsilon_{yx}-\upsilon_{yz}\upsilon_{zy}-\upsilon_{zx}\upsilon_{xZ}-2\upsilon_{yz}\upsilon_{zy}\upsilon_{xz}>0$$

Unidirectional layer of composite

- Thin composite structures
  - neglecting through thickness stresses – plane stress model
  - enable to simplify the model for composite laminates

orthotropic material model

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\upsilon_{21}/E_2 & -\upsilon_{31}/E_3 & 0 & 0 & 0 \\ -\upsilon_{12}/E_1 & 1/E_2 & -\upsilon_{32}/E_3 & 0 & 0 & 0 \\ -\upsilon_{13}/E_1 & -\upsilon_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}$$

Parameters:
E_x, E_y, E_z, G_{xy}, G_{xz}, G_{yz}, ν_{xy}, ν_{xz}, ν_{yz}

lamina material model (plane stress)

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\upsilon_{21}/E_2 & 0 \\ -\upsilon_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix}$$

Parameters:
E_x, E_y G_{xy}, ν_{xy}, (G_{xz}, G_{yz})

- Although 4 parameters are necessary (E_x, E_y, G_{xy}, ν_{xy}), the other two shear modules should be included as well
  - due to low values of shear modulus of fibre composites
  - to prevent unreasonable deformations of finite element model

Unidirectional layer of composite

- Modelling of UD layer
  - thickness
  - material properties (E_x, E_y, G_{xy}, ν_{xy},G_{xz}, G_{yz})
  - **material orientation**
    - Abaqus – orientation must be specified for not isotropic material; otherwise input will not pass solver check
    - Ansys APDL – if not specified, orientation is taken from the global coordinate system**
- Elements
  - Shell elements
  - Solid elements (full orthotropic material model needed)
    - be careful for the transverse shearing stresses
  - Beams

- In reality, most composite structures compose of layers (UD or bi-directional) with various orientation

** for shell elements the situation is more complicated

Unidirectional layer of composite

- Mechanical properties of layer
  - Necessity to input E_x, E_y, G_{xy}, ν_{xy}, G_{xz}, G_{yz}
  - How to get these constants?
    - From the manufacturer of semi-finished product (prepregs)
    - From experimental measurements
    - By computation from fibre and matrix properties and assumed fibre volume fraction (rule of mixture, micromechanics of composites)
  - Issues
    - Parameters of fibres can be unknown (mostly parameters in transverse direction)
    - Micromechanical model or rule of mixture might not correspond to the selected fibre
      - Different models for isotropic fibres (glass) and orthotropic fibres
      - Variation between the models and experimental behaviour
    - Theoretical fibre volume fraction does not match with the fibre volume fraction of real composite component
    - Different tensile and compressive modulus Ex of carbon fibre composites (approx. 10%)

Unidirectional layer of composite

- Mechanical properties of layer
  - Necessity to input E_x, E_y, G_{xy}, ν_{xy}, G_{xz}, G_{yz}
  - Example of rule of mixture
    - presented equations the most simple, not necessary the most accurate

- Longitudinal modulus of layer

$$E_L = V_f E_f + \left(1 - V_f\right) E_m$$

- Transverse modulus of layer

$$E_T = \frac{E_m}{1 - V_f \left(1 - \frac{E_m}{E_f}\right)} \approx \frac{E_m}{1 - V_f}$$

- In-plane Poisson number

$$\nu_{LT} = V_f \nu_f + V_m \nu_m$$

- In-plane Shear modulus

$$G_{LT} = \frac{G_m}{1 - V_f \left(1 - \frac{G_m}{G_f}\right)} \approx \frac{G_m}{1 - V_f}$$

Unidirectional layer of composite

- Mechanical properties of layer
  - Necessity to input E_x, E_y, G_{xy}, ν_{xy}, G_{xz}, G_{yz}
  - Should the factors like the fibre properties be included in the selection of the mechanical model?

- Transverse modulus of layer*

$$E_{\mathrm{T}} = \frac{E_m}{1 - \left(1 - \frac{E_m}{E_{fT}}\right) . \sqrt{V_f}}$$

- Transverse modulus of layer

$$E_T = \frac{E_m}{1 - V_f \left(1 - \frac{E_m}{E_f}\right)} \approx \frac{E_m}{1 - V_f}$$

- In-plane Shear modulus*

$$G_{\mathrm{LT}} = \frac{G_m}{1 - \sqrt{V_f} . \left(1 - \frac{G_m}{G_{fLT}}\right)}$$

- In-plane Shear modulus

$$G_{LT} = \frac{G_m}{1 - V_f \left(1 - \frac{G_m}{G_f}\right)} \approx \frac{G_m}{1 - V_f}$$

* Equations – Chamis model, CHAMIS, Christos C. *Simplified Composite Micromechanics Equations for Strength, Fracture Toughness and Environmental Effects.* Houston, January 1984. Report No. NASA TM-83696. National Aeronautics and Space Administration.

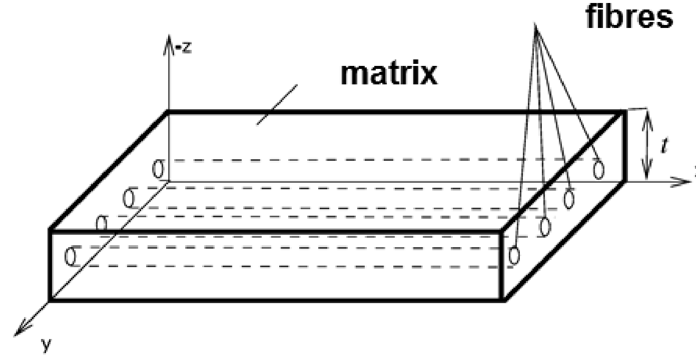
Unidirectional layer of composite

- Mechanical properties of layer
  - E_x, E_y, G_{xy}, ν_{xy}, G_{xz}, G_{yz}
  - How to calculate other parameters E_Z, ν_{xz}, ν_{yz}, G_{xz}, G_{yz} ?

$E_x = E_L$

$E_y = E_z = E_T$

$G_{xy} = G_{xz} = G_{LT}$



  - G_{yz}, ν_{yz} – quite problematic

Tsai (A):

$$G_{23} = \frac{V_f + \delta(1-V_f)}{V_f / G_{f23} + \delta(1-V_f) / G_m}$$

$$\delta = \frac{3-4\upsilon_m + G_m / G_f}{4(1-\upsilon_m)}$$

Chamis (B):

$$G_{23} = \frac{G_m}{1-\sqrt{V}\,(1-G_m / G_{f23})}$$

$G_{f23} > G_m$

Hashin

$$G_{23}^{(-)} = G_m\left[1+\frac{V_f}{\frac{G_m}{G_{f23}-G_m} + (1-V_f)\frac{3K_m+7G_m}{6K_m+8G_m}}\right]$$

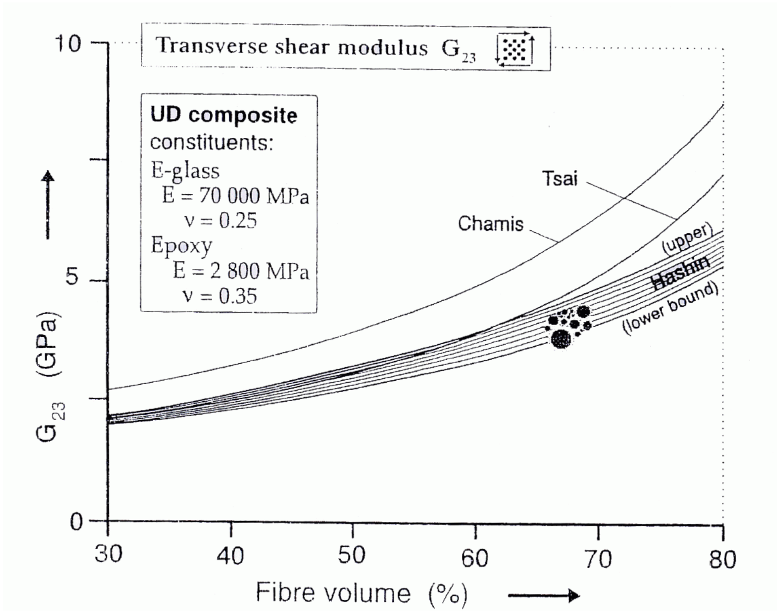
$$K_m = \frac{E_m}{3(1-2\upsilon_m)}$$

$$G_{23} = \frac{E_2}{2(1+\nu_{23})} = \frac{E_3}{2(1+\nu_{32})}$$

Unidirectional layer of composite

- Mechanical properties of layer
  - E_x, E_y, G_{xy}, ν_{xy}, G_{xz}, G_{yz}
  - How to calculate other parameters E_z, ν_{xz}, ν_{yz}, G_{xz}, G_{yz} ?

$E_x = E_L$
$E_y = E_z = E_T$
$G_{xy} = G_{xz} = G_{LT}$



  - G_{yz}, ν_{yz} – quite problematic

Tsai (A):

$$G_{23} = \frac{V_f + \delta(1 - V_f)}{V_f / G_{f23} + \delta(1 - V_f) / G_m}$$

$$\delta = \frac{3 - 4\upsilon_m + G_m / G_f}{4(1 - \upsilon_m)}$$

Chamis (B):

$$G_{23} = \frac{G_m}{1 - \sqrt{V}\left(1 - G_m / G_{f23}\right)}$$

$G_{f23} > G_m$

Hashin

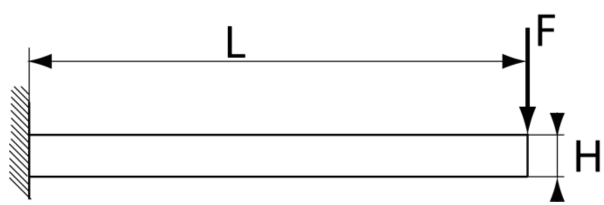
$$G_{23}^{(-)} = G_m \left[1 + \frac{V_f}{\frac{G_m}{G_{f23} - G_m} + \left(1 - V_f\right) \frac{3K_m + 7G_m}{6K_m + 8G_m}} \right]$$

$$K_m = \frac{E_m}{3(1 - 2\upsilon_m)}$$

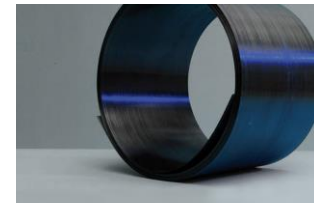
$$G_{23} = \frac{E_2}{2(1 + \nu_{23})} = \frac{E_3}{2(1 + \nu_{32})}$$

Unidirectional layer of composite

- Effect of transverse shearing
  - Bending of rectangular beam



$$u = \underbrace{\frac{F.L^3}{3(EJ)}}_{\text{bending}} + \underbrace{\frac{\beta.F.L}{(GA)}}_{\text{transverse shearing}}$$

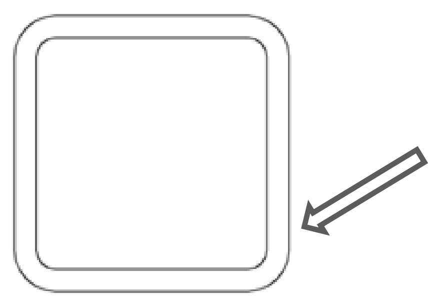


Beam of rectangular cross-section

- (EJ) – modulus E_1
- (GA) – modulus G_{13}

Material	ρ_f [kg.m^{-3}]	E_1 [GPa]	G_{13} [GPa]
steel	7850	210	80
uhm/E	1750	380	3 (2÷4)

- For orthotropic beam profile low stiffness in transverse shearing
  - can be neglected when length/thickness ratio is 30 (20) and more
  - increase of thickness not efficient, need to change material orientation



combination of layers 0 and [45,-45]s

Layered structures & Laminates

- Let's get to reality

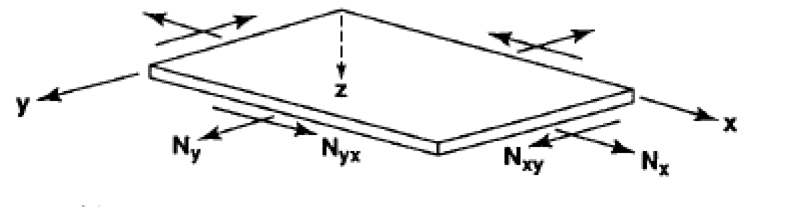
Layered structures & Laminates

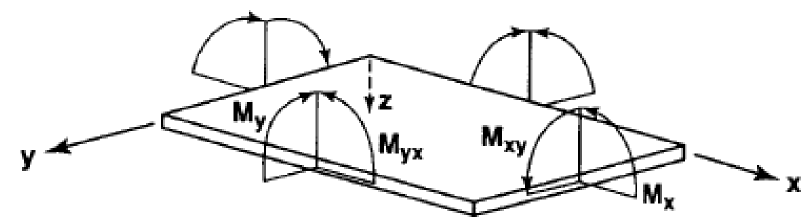
- Real composite components – composed of layers with different orientations
  - Hand made laminates
  - Laminates from prepregs
  - RTM products (resin transfer moulding)
  - Filament or tape winding products, braiding

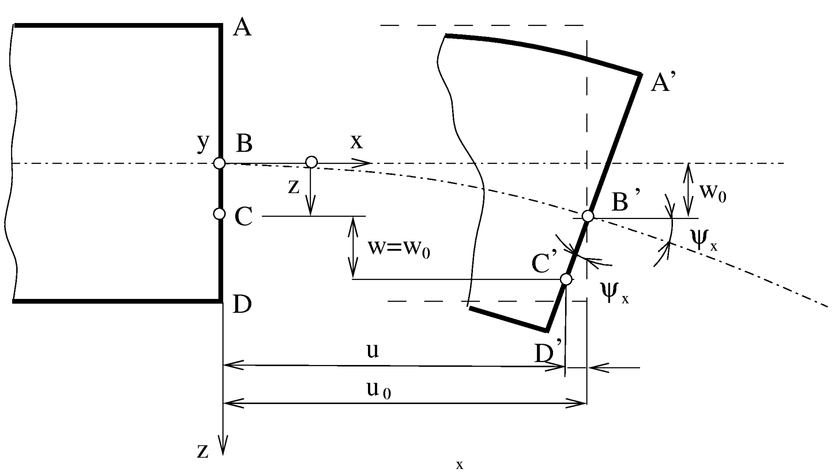
- In comparison with isotropic FE models
  - Restriction of element types
  - More time consuming preprocessing of the model
  - More data consuming model
  - Need to have clear idea what to do at the beginning of pre-processing
  - Simplifications necessary, but might lead to fatal errors in modelling or post-processing

Layered structures – Laminate theory

- Classical laminate theory
  - relations between the load and deformations of the laminate
  - plane stress state in the laminate
  - neglecting transverse shear stresses
  - thickness of layer is significantly smaller than other dimensions
  - rigid interference between the layers





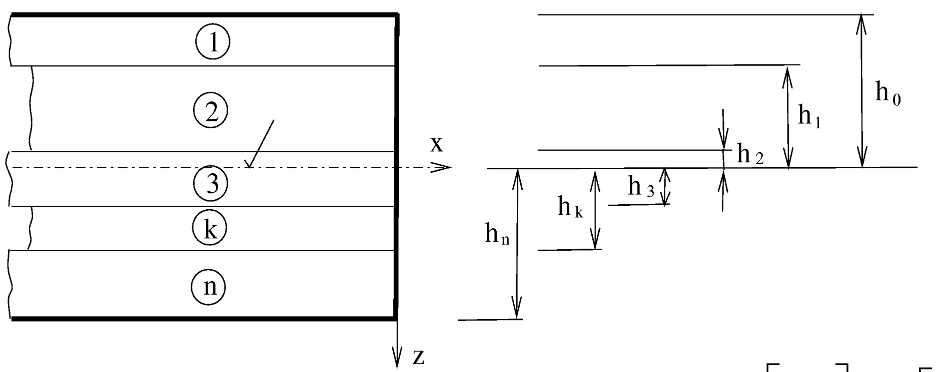


$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

Layered structures – Laminate theory

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

- Properties of laminate can be described by ABD matrix
- In general, ABD matrix contains all components



$$A_{ij} = \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(h_k - h_{k-1}\right)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(h_k^2 - h_{k-1}^2\right)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(h_k^3 - h_{k-1}^3\right)$$

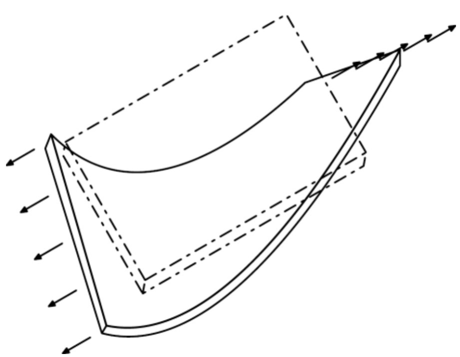
$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} + z \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

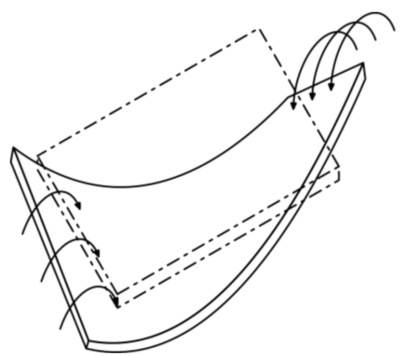
Layered structures – ABD matrix and its meaning

- Full ABD matrix
  - 1 loading component leads to all deformation effects
    - normal strains
    - bending strains
    - twisting strains
    - shearing strains
  - The coupled deformation effect might cause problems when simplifying modelling
    - **using the symmetry of models**
    - **using the homogenized material constants**

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} & \overline{A}_{16} & \overline{B}_{11} & \overline{B}_{12} & \overline{B}_{16} \\ \overline{A}_{21} & \overline{A}_{22} & \overline{A}_{26} & \overline{B}_{21} & \overline{B}_{22} & \overline{B}_{26} \\ \overline{A}_{61} & \overline{A}_{62} & \overline{A}_{66} & \overline{B}_{61} & \overline{B}_{62} & \overline{B}_{66} \\ \overline{B}_{11} & \overline{B}_{12} & \overline{B}_{16} & \overline{D}_{11} & \overline{D}_{12} & \overline{D}_{16} \\ \overline{B}_{21} & \overline{B}_{22} & \overline{B}_{26} & \overline{D}_{21} & \overline{D}_{22} & \overline{D}_{26} \\ \overline{B}_{61} & \overline{B}_{62} & \overline{B}_{66} & \overline{D}_{61} & \overline{D}_{62} & \overline{D}_{66} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$





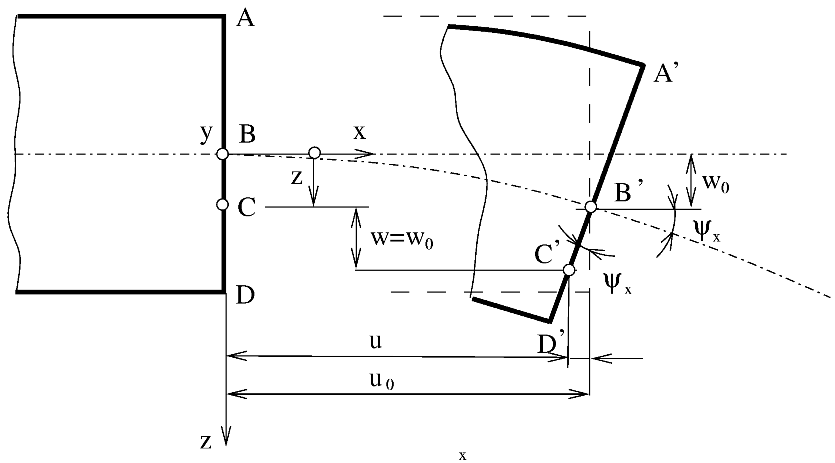
Layered structures – ABD matrix and its meaning

- Effect of composite lay-up on ABD matrix

	$[\mathbf{A}]$	$[\mathbf{B}]$	$[\mathbf{D}]$	
Symmetric	$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$	$[45 \; \lvert 90 \rvert \; 0 \; \lvert 60 \rvert -30]_S$
Balanced	$\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$	$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$	$[30 \; \lvert -60 \rvert \; 0 \; \lvert 60 \rvert -30]$
Symmetric balanced	$\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$	$[30 \; \lvert -30 \rvert \; 60 \; \lvert -60]s$
Symmetric cross-ply	$\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & O \\ 0 & 0 & D_{66} \end{bmatrix}$	$: \; [0 \;\; 90 \;\; 0_2 \;\; 90 \;\; 0 \;\;]$
Antisymmetric cross-ply	$\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$	$\begin{bmatrix} B_{11} & 0 & 0 \\ 0 & -B_{11} & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & O \\ 0 & 0 & D_{66} \end{bmatrix}$	$[0 \;\; 90 \;\; 0 \;\; 90 \;\; 0 \;\; 90 \;\;]$

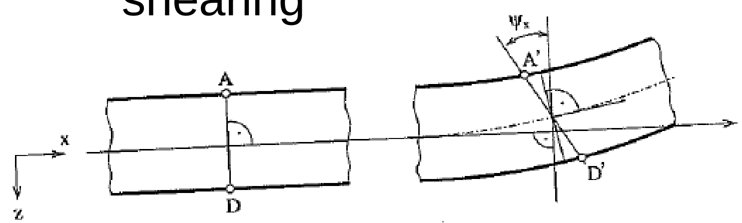
Layered structures – first order shear theory

- Classical laminate theory
  - Kirchhoff



$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

- First order shear theory
  - with effect of transverse shearing



$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_y \\ Q_x \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & 0 & 0 \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & 0 & 0 \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{44} & F_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{45} & F_{55} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma^{\circ}_{yz} \\ \gamma^{\circ}_{xz} \end{bmatrix}$$

Layered structures – first order shear theory

- Transverse shearing
  - can be neglected for very thin plates
  - for composites, the length to thickness ratio, from which it is possible to neglect transverse shearing, is significantly higher than for isotropic materials
  - FEA – shells generally with FOST

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_y \\ Q_x \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & 0 & 0 \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & 0 & 0 \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{44} & F_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{45} & F_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx}^{\circ} \\ \varepsilon_{yy}^{\circ} \\ \gamma_{xy}^{\circ} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{yz}^{\circ} \\ \gamma_{xz}^{\circ} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{26} & 0 & 0 \\ Q_{61} & Q_{62} & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & C'_{44} & C'_{45} \\ 0 & 0 & 0 & C'_{54} & C'_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz}^{0} \\ \gamma_{xz}^{0} \end{bmatrix}$$

$$F_{ij} = \sum_{k=1}^{n} \left(C'_{ij}\right)_k \left(h_k - h_{k-1}\right), \quad i, j = 4,5$$

Layered structures – "homogenization"

$$\begin{bmatrix} \varepsilon_{xx}^{\circ} \\ \varepsilon_{yy}^{\circ} \\ \gamma_{xy}^{\circ} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{16} & \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} \\ \bar{A}_{21} & \bar{A}_{22} & \bar{A}_{26} & \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} \\ \bar{A}_{61} & \bar{A}_{62} & \bar{A}_{66} & \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} \\ \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} & \bar{D}_{11} & \bar{D}_{12} & \bar{D}_{16} \\ \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} & \bar{D}_{21} & \bar{D}_{22} & \bar{D}_{26} \\ \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} & \bar{D}_{61} & \bar{D}_{62} & \bar{D}_{66} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$

- Inverse matrix to ABD can be used for determination of equivalent material properties of the laminate
  - E_x, E_y, G_{xy}, ν_{xy}
  - This approach leads to simplified modelling, but with reduced accuracy (missing the coupling between the deformations)
  - Homogenized constants might violate the stability conditions of material model
  - ABD more precise

$$\begin{bmatrix} \varepsilon_{xx}^{\circ} \\ \varepsilon_{yy}^{\circ} \\ \gamma_{xy}^{\circ} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{16} & \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} \\ \bar{A}_{21} & \bar{A}_{22} & \bar{A}_{26} & \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} \\ \bar{A}_{61} & \bar{A}_{62} & \bar{A}_{66} & \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} \\ \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} & \bar{D}_{11} & \bar{D}_{12} & \bar{D}_{16} \\ \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} & \bar{D}_{21} & \bar{D}_{22} & \bar{D}_{26} \\ \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} & \bar{D}_{61} & \bar{D}_{62} & \bar{D}_{66} \end{bmatrix} \begin{bmatrix} N_x \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{N_x}{A.E_{x\_tah}} = \varepsilon_{xx}^{0} = \bar{A}_{11}.N_x$$

$$E_{x\_tah} = \frac{1}{\bar{A}_{11}.\sum t_i}$$

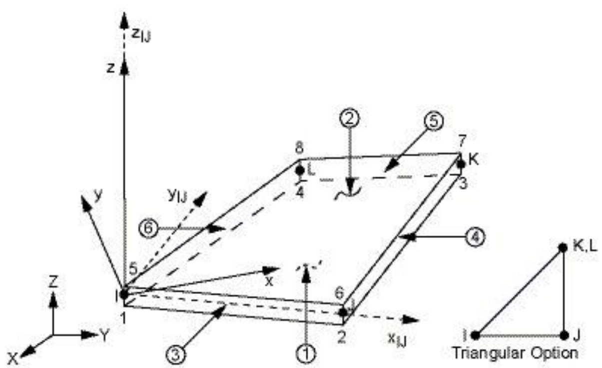
$$\begin{bmatrix} \varepsilon_{xx}^{\circ} \\ \varepsilon_{yy}^{\circ} \\ \gamma_{xy}^{\circ} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{16} & \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} \\ \bar{A}_{21} & \bar{A}_{22} & \bar{A}_{26} & \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} \\ \bar{A}_{61} & \bar{A}_{62} & \bar{A}_{66} & \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} \\ \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} & \bar{D}_{11} & \bar{D}_{12} & \bar{D}_{16} \\ \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} & \bar{D}_{21} & \bar{D}_{22} & \bar{D}_{26} \\ \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} & \bar{D}_{61} & \bar{D}_{62} & \bar{D}_{66} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ M_x \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{M_o(x)}{E_{x\_ohyb}.J} = k_x = \bar{D}_{11}.M_x$$

$$E_{x\_ohyb} = \frac{1}{12.\bar{D}_{11}.(\sum t_i)^3}$$

Elements for FEA of composite structures

- Ansys FE solver – recommended elements
  - **layered shell elements (shell181, shell 281)**
  - layered solid-shell elements (solidshell 190)
  - layered solid elements (solid185, solid186),
  - beam elements



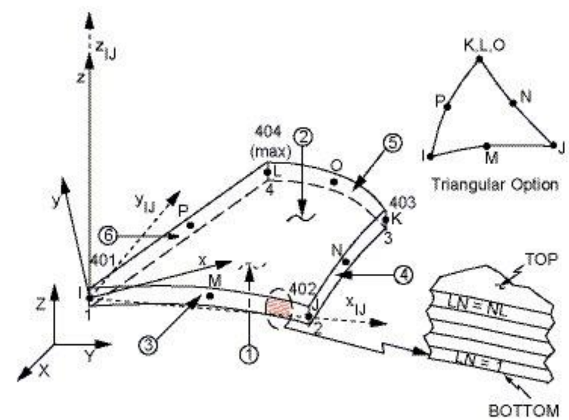
x_{IJ} = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

4-node layered shell element **SHELL181**

- modelled on reference surface
- each node 3 translation DOF and 3 rotation DOF

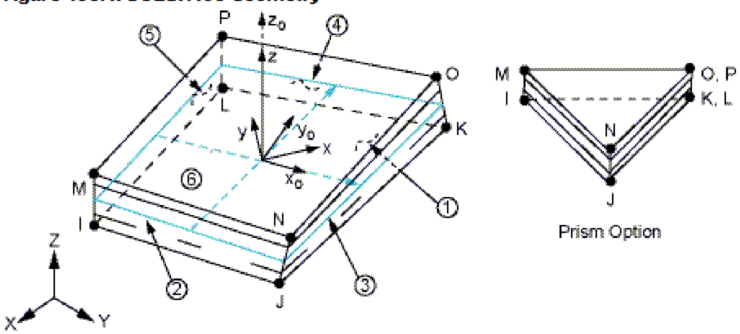
8-node layered shell element
SHELL281 (~~SHELL91~~, ~~SHELL99~~)



Elements for FEA of composite structures

- Ansys FE solver – recommended elements
  - layered shell elements (shell181, shell 281)
  - **layered solid-shell elements (solidshell 190)**
  - layered solid elements (solid185, solid186),
  - beam elements

Figure 190.1: SOLSH190 Geometry



x_o = Element x-axis if ESYS is not supplied.

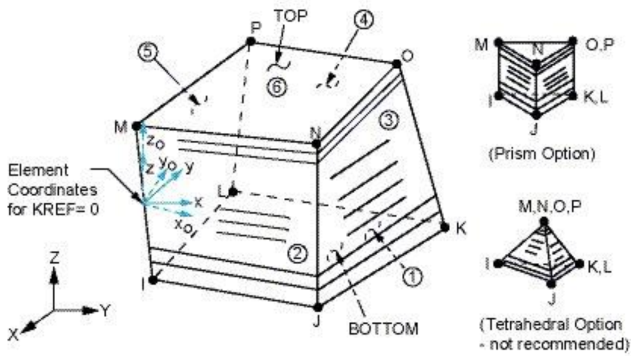
x = Element x-axis if ESYS is supplied.

8-node solid-shell element **SOLSH190**

- Solid geometry, node – 3 translation DOFS
- Behaviour similar to shell elements
- It is necessary to have consistent orientation of element in thickness direction
  - VEORIENT
  - EORIENT
- For thicker components more precise than classical shells, can be stacked through thickness
- In comparison with solids more precise in transverse sharing stresses

Elements for FEA of composite structures

- Ansys FE solver – recommended elements
  - layered shell elements (shell181, shell 281)
  - **layered solid-shell elements (solidshell 190)**
  - layered solid elements (solid185, solid186),
  - beam elements



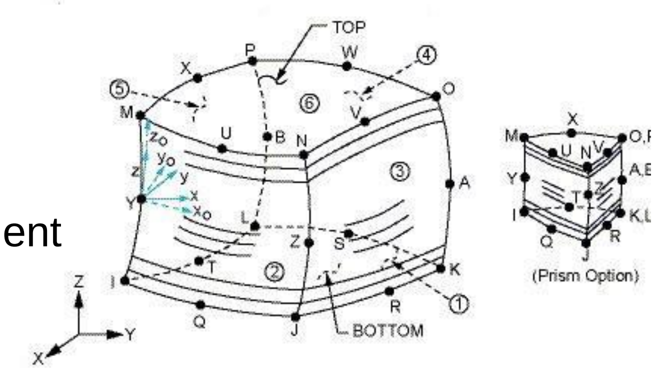
x_o = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

8-node layered solid element **SOLID185**

- limited usage (free edge problems,…)

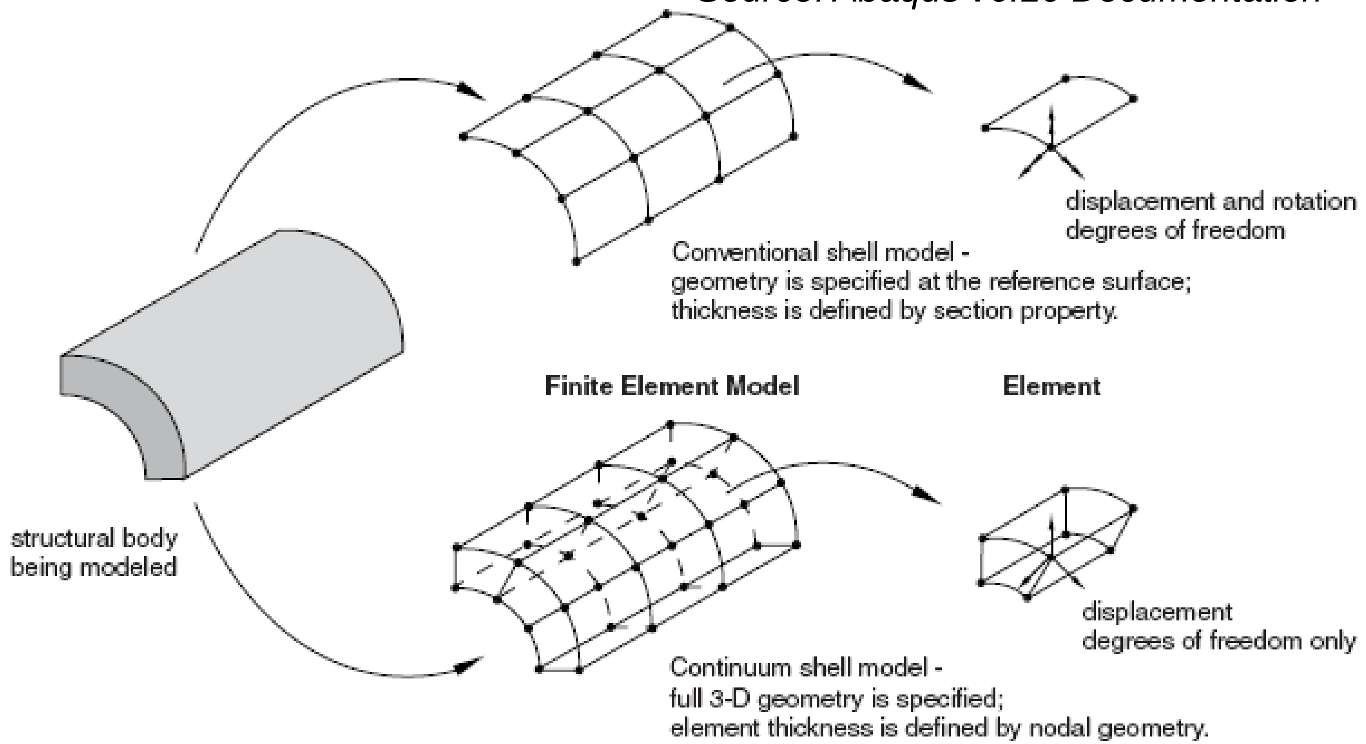
20-node layered solid element **SOLID186**



Elements for FEA of composite structures

- Abaqus & composites
  - *solid elements*
  - conventional shell elements
  - continuum shell elements (solid-shell elements from previous slides)

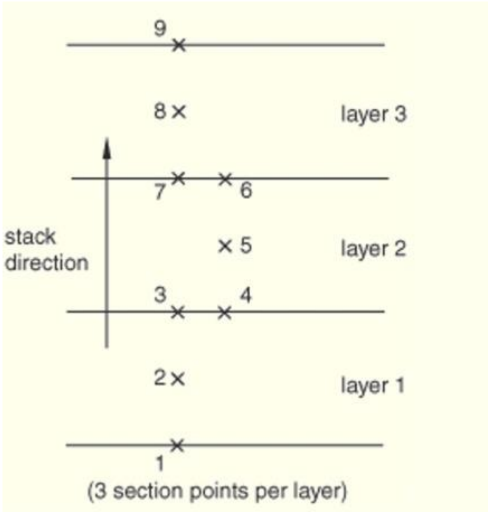
Source: Abaqus v6.10 Documentation



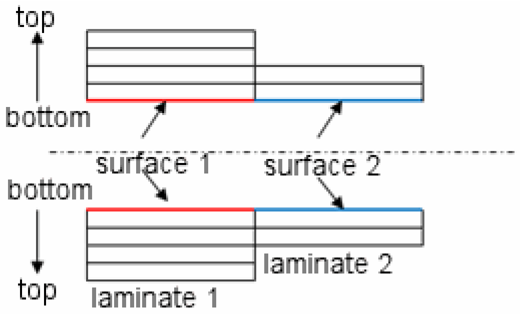
Elements for FEA of composite structures - Shells

- **Conventional shells**
  - The most common elements for modelling of components from fibre reinforced plastics
  - Enable to easily define the material, orientation and thickness of every layer of lay-up
  - Layers are modelled in the same order as were defined, stacking is in direction of shell normal
    - the first layer is at the bottom of shell
    - the last layer at the top surface of shell
  - Results of shell in integration points, in every layer section points through thickness
  - Shell are modelled on the reference surface
    - Reference surface – on midsurface
    - Reference surface - offset from the midsurface
      - – enabling to model the ply-drops

Source Abaqus v6.10 Documentation



Source Solidworks help



Elements for FEA of composite structures - Shells

- **Basic assumptions for using shell elements**
  - each ply is modelled as homogenous, its thickness is significantly smaller in comparison with the other dimensions
  - interface between the layers is ideally rigid, thin, the displacements of the layers through the interfaces are therefore continuous
  - Kirchhof or First Order Shear Theory
  - shell thickness does not change with deformation
  - the ration of smallest dimension of shell surface to its thickness is larger than 10
  - stiffness of laminate in coordinates X, Y, Z of shell does not differ by more than 2 orders (might be violated in sandwich constructions)
  - more: http://mechanika2.fs.cvut.cz/old/pme/predmety/mkp1/podklady/skorepiny_ju.pdf

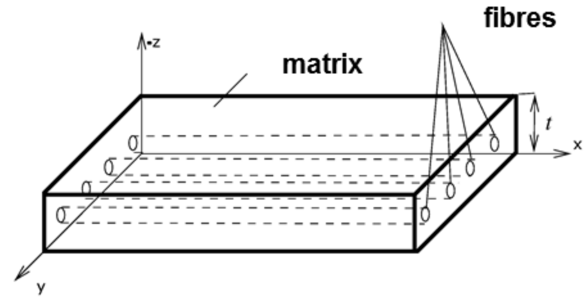
Elements for FEA of composite structures - Shells

- Basic difference in comparison with modelling of isotropic materials
  - Potential source of fatal errors if neglected
- Isotropic shells in commercial FE solvers
  - default: data stored in the top and bottom layer of the shell
    - maximum of bending stresses
    - safe for evaluation of strength
- Orthotropic shells
  - when using default settings without enhancing the data storage to every layer
    - layers with maximal loading might be not evaluated in terms of stress, strain and failure
    - only top and bottom layer post-processed
- Works both for conventional and continuum shells
  - If you need to investigate the stress loading of component and potential failure, you need to know the stress loading of every layer in critical area of components
    - If deformations are needed only, this can be neglected

Strength evaluation

- Orthotropic materials
  - strength differs for different modes of loading
  - do not evaluate by isotropic approaches (von Mises stress, …)

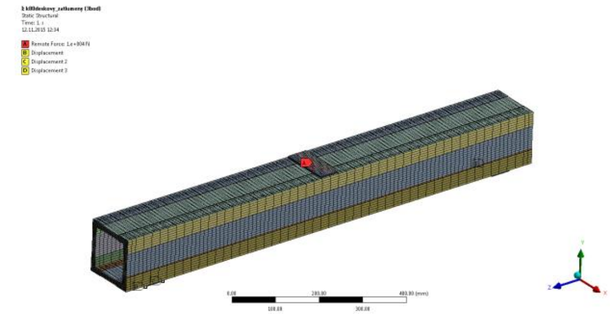
- FEA of composite structure
  - failure index for the first ply failure
    - maximal stress or strain criterion
    - Tsai-Wu, Tsai-Hill, …
    - PUCK, LARC03,LARC04
    - User defined criteria
  - might be complicated to get all data of ply strengths for the criteria evaluation
  - not every criteria is suitable for the loading mode (but still better to be used than to use von Mises stress)

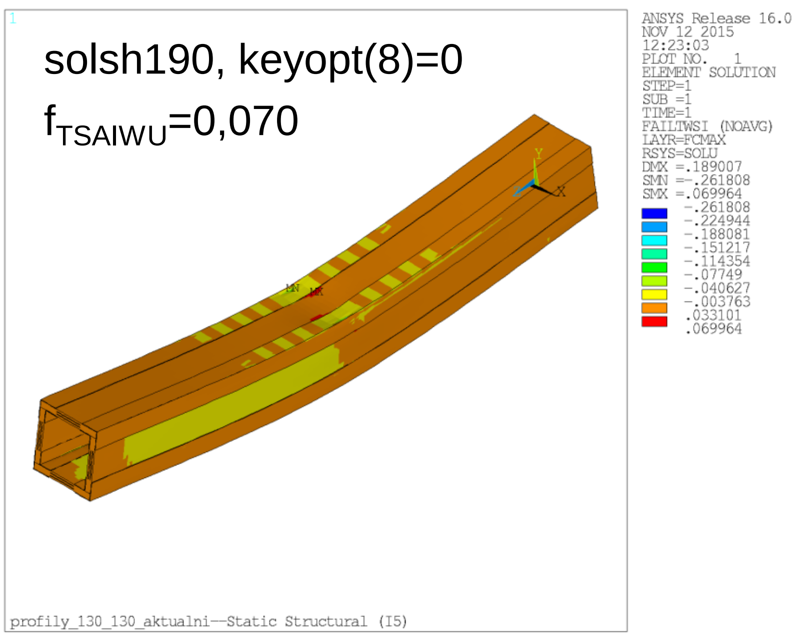


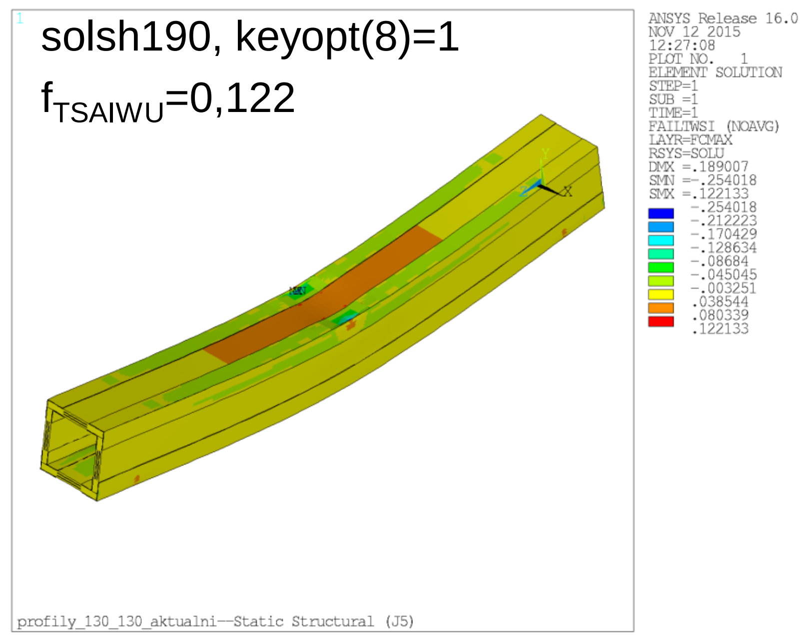
		AS4/E	E-glass/E
V_f	[%]	60	62
X_T	[MPa]	1950	1140
X_c	[MPa]	1480	900
Y_T	[MPa]	48	35
Y_c	[MPa]	200	114
S_{12}	[MPa]	79	72
ε_{1T}	[%]	1,38	2,13
ε_{1C}	[%]	1,18	1,07
ε_{2T}	[%]	0,44	0,20
ε_{2C}	[%]	2,0	0,64
ε_{12}	[%]	2	3,8

Strength evaluation

- Failure index f
  - f<1 – no first ply failure
  - f=1 – first ply failure
- Do not forget to evaluate data through all the layers specified in the lay (i.e. not only from the top and bottom layer)



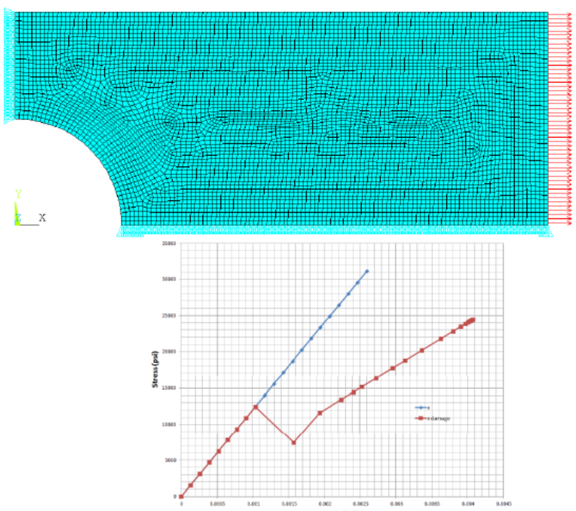


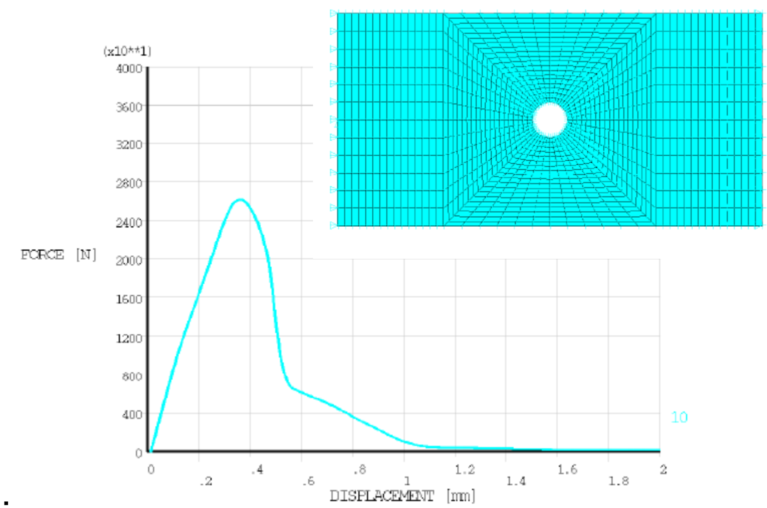


Strength evaluation

- Options to model progressive damaging of composites
  - Stiffness degradation due to damage initiation and growth
  - Abaqus, Ansys - Options for progressive damage implemented

Source CAE Associates – Progressive Damage of Fiber – Reinforced Composites in Ansys v15





- Options to investigate the composites delamination
  - Cohesive Zone Modelling
  - Virtual Crack Closure Technique
  - used also for simulations of debonding of adhesive joints between the components

Composite structures

- Short conclusions in terms of modelling – structural level
  - Usually thin components (thickness is significantly smaller than other 2 dimensions)
    - Suitable for shell elements, beam elements
    - Options for solid modelling limited
  - Usually composite lay-up with layers with multiangle orientations, structures with only 1 orientation of fibres are rare
    - Conventional shell elements
      - definition of full composite lay-up
        - material, thickness, orientation in respect to element normal
      - specification by ABD matrix
        - ABD matrix, optionally with transverse shear stiffness
      - specification by homogenized properties
        - modules of laminate

Composite structures

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  - Usually thin components (thickness is significantly smaller than other 2 dimensions)
    - Suitable for shell elements, beam elements
    - Options for solid modelling limited
  - Usually composite lay-up with layers with multiangle orientations, structures with only 1 orientation of fibres are rare
    - Continuum shell elements
      - definition of full composite lay-up
        - material, relative thickness, orientation in respect to element normal
      - specification by homogenized properties
        - modules of laminate
      - specification by ABD matrix not applicable
      - must be divided into sub-laminates if having more than 1 element through thickness

Composite structures

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      - definition of full composite lay-up
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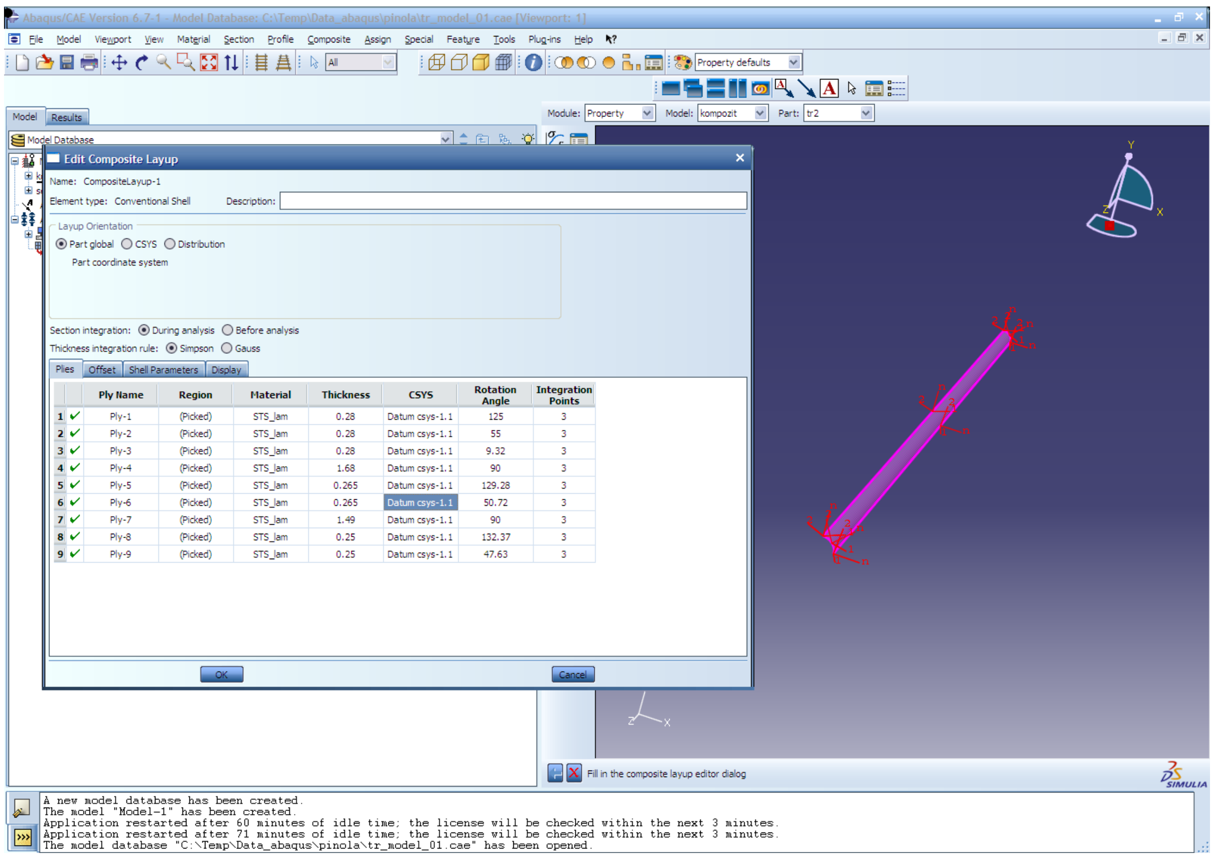
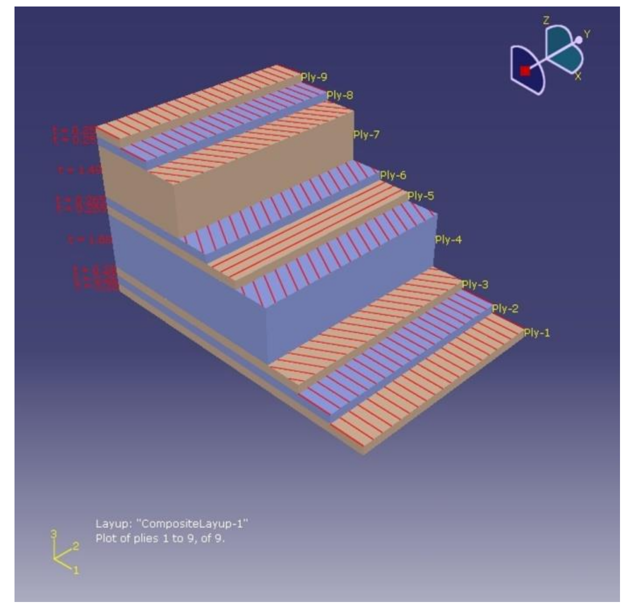
      - specification by homogenized properties
        - » modules of laminate
      - specification by ABD matrix not applicable
      - must be divided into sub-laminates if having more than 1 element through thickness

Composite structures

- Lay-up specification
  - Abaqus
    - shell section
    - composite lay-up manager (preferable)

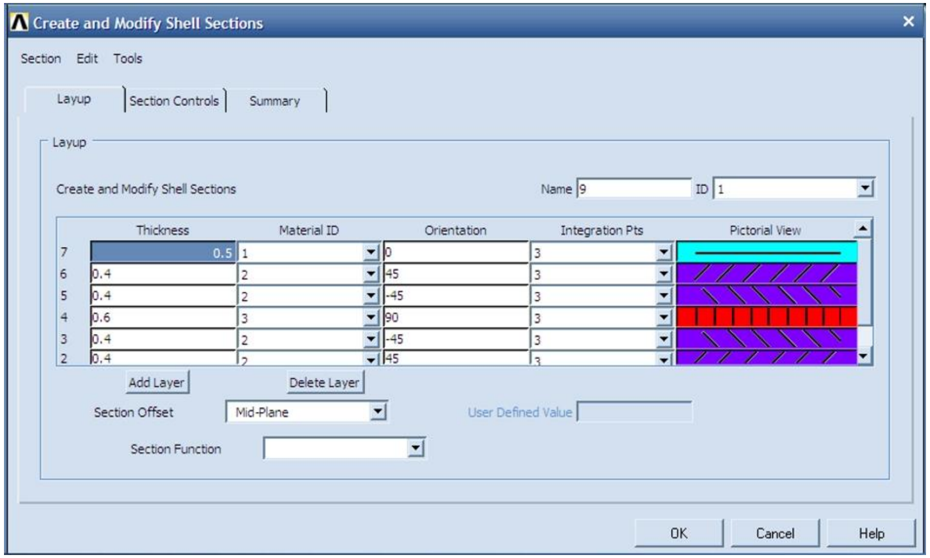


	Ply Name	Region	Material	Thickness	CSYS	Rotation Angle	Integration Points
1	Ply-1	(Picked)	STS_lam	0.28	Datum csys-1.1	125	3
2	Ply-2	(Picked)	STS_lam	0.28	Datum csys-1.1	55	3
3	Ply-3	(Picked)	STS_lam	0.28	Datum csys-1.1	9.32	3
4	Ply-4	(Picked)	STS_lam	1.68	Datum csys-1.1	90	3
5	Ply-5	(Picked)	STS_lam	0.265	Datum csys-1.1	129.28	3
6	Ply-6	(Picked)	STS_lam	0.265	Datum csys-1.1	50.72	3
7	Ply-7	(Picked)	STS_lam	1.49	Datum csys-1.1	90	3
8	Ply-8	(Picked)	STS_lam	0.25	Datum csys-1.1	132.37	3
9	Ply-9	(Picked)	STS_lam	0.25	Datum csys-1.1	47.63	3



Composite structures

- Lay-up specification
  - Ansys
    - shell section (Mechanical APDL, Workbench through APDL commands)
    - Ansys Composite Pre-Post
      - graphical interface for composite materials
      - additional plug-in to Ansys, available to students of CTU in Prague

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secdata,0.4,2,45,3
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secdata,0.6,3,903
secdata,0.4,2,-45,3
secdata,0. 4,2,45,3
secdata,0.5,2,0,3
!
et,2,solsh190
emodif,all,type,2
emodif,all,esys,11
emodif,all,secnum,1
```

Example 1

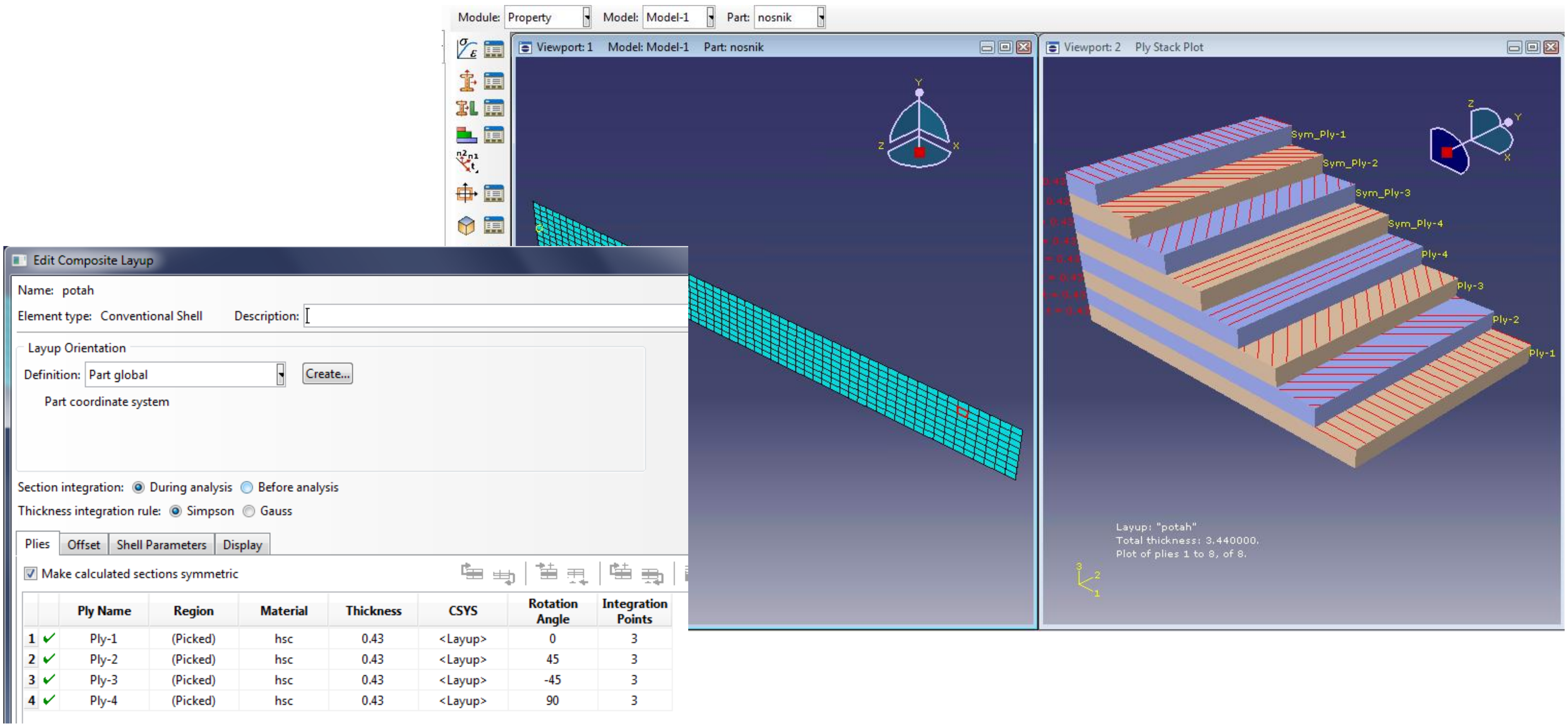
- Laminate beam
  - Laminate from UD prepregs
  - Dimensions 70x700 mm
  - Lay-up [0, 45, -45, 90]s
    - high-strength C/E
    - high-modulus C/E
  - material data
    - from prepreg manufacturer sheets
    - additional parameters from micromechanics

Mechanical Properties

	T700 UD HS Carbon Fibre	RC200T	RC200T
Resin System	SE 84LV	SE 84LV	SE 84LV
Cure (time / temperature / pressure)	10 hrs / 85°C / 1 Bar	10 hrs / 85°C / 1 Bar	1 hr / 120°C / 6 Bar
Process	vacuum bag	vacuum bag	press
Fibre Weight (g/sqm)	300	194	194
Prepreg Areal Weight (g/sqm)	476	334	334
Prepreg Resin Content (%bw)	37	42	42
Tensile Strength (MPa)	2844	760	1074
Tensile Modulus (GPa)	129.2	55.9	66.4
Tensile Laminate Fibre Vol. (%)	59.8	56.6	60.8
Cured Ply Thickness** (mm)	0.281	0.214	0.199
Normalised Tensile Strength @ 60% FVF (MPa)	2854	806	1060
Normalised Tensile Mod. @ 60% FVF (GPa)	129.7	65.2	65.4
Compressive Strength (MPa)	1187	718	767
Compressive Laminate Fibre Volume (%)	57.5	56	60.3
Normalised Compr. Strength @ 60% FVF (MPa)	1239	770	764
ILSS (MPa)	79	76	70

Example 1

- Laminate beam
  - Shell finite element model
  - Full Lay-up specification

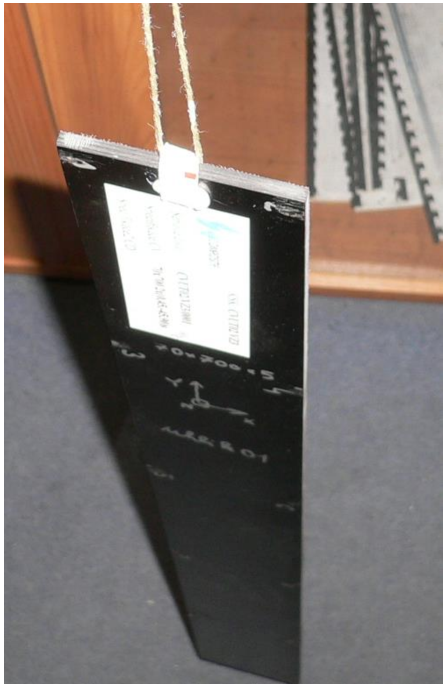


	Ply Name	Region	Material	Thickness	CSYS	Rotation Angle	Integration Points
1	Ply-1	(Picked)	hsc	0.43	<Layup>	0	3
2	Ply-2	(Picked)	hsc	0.43	<Layup>	45	3
3	Ply-3	(Picked)	hsc	0.43	<Layup>	-45	3
4	Ply-4	(Picked)	hsc	0.43	<Layup>	90	3

Example 1

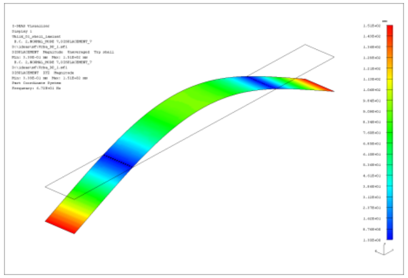
- Comparison with experimental results
  - modal analysis
    - mode shapes and its frequencies
    - match between FE and experiment acceptable

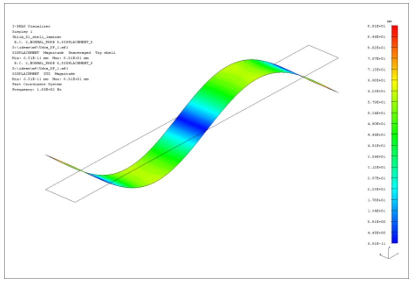
Mode [-]	Experiment [Hz]	FEA [Hz]
1	42.5	46.4
2	121.5	132.6
3	193.5	206.2
4	242.4	266.4
5	406.2	419.4
Mass [g]	262.5	263.8

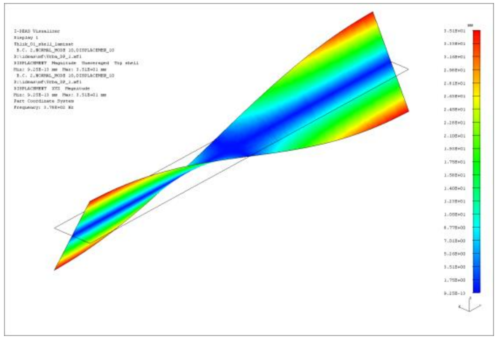


Example 1

- Comparison with experimental results
  - laminate beam from HM/E UD prepregs [0, 45, -45, 90]s
  - laminate modelled by
    - ABD 1 matrix
    - homogenized E_x, E_y, G_{xy}, ν_{xy}, G_{xz}, G_{yz} of the lay-up
    - ABD 2 matrix with transverse shear stiffness (ABDF)
  - using first order shear theory with specified transverse stiffness most precise





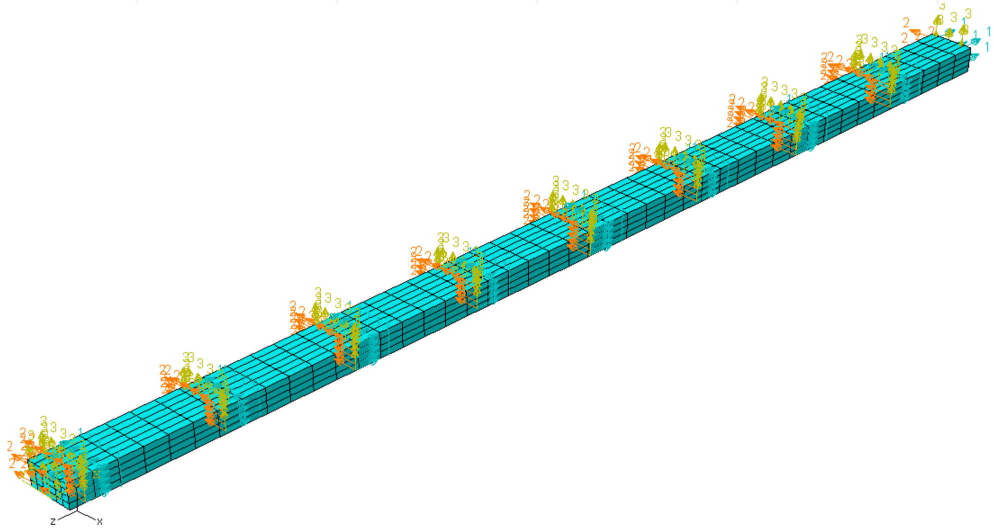


Mode [-]	Exp. [Hz]	ABD1 [Hz]	hom. const [Hz]	ABDF [Hz]
bend.	68.1	73.7	67.2	68.2
bend	193	203	185	188
tors.	331	371	378	346
bend	380	398	363	368
bend.	626	656	599	608
tors.	686	748	762	697

Example – Unidirectional beam

- Unidirectional thick-walled beam
  - beam 740x30x20
  - material: ultra-high modulus carbon / epoxy composite
  - modelled by solid elements C3D8I (Abaqus)
  - material properties
    - from fibre and matrix parameters, assumed fibre volume fraction
    - estimation $\nu 23$, G23

E1	E2	E3	Nu12	Nu13	Nu23	G12	G13	G23
390000	3550	3550	0.34	0.34	0.4	3000	3000	3000



Experiment [Hz]	FEA [Hz]
590.2	596.8
833.1	797.0
893.3	836.7
1457.7	1442.0
1804.1	1610.4
1873.1	1821.0

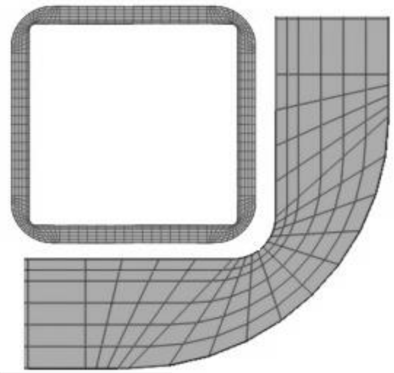
  - first bending mode shapes with good precision; torsional mode more inaccurate

Example – beam profile coupons

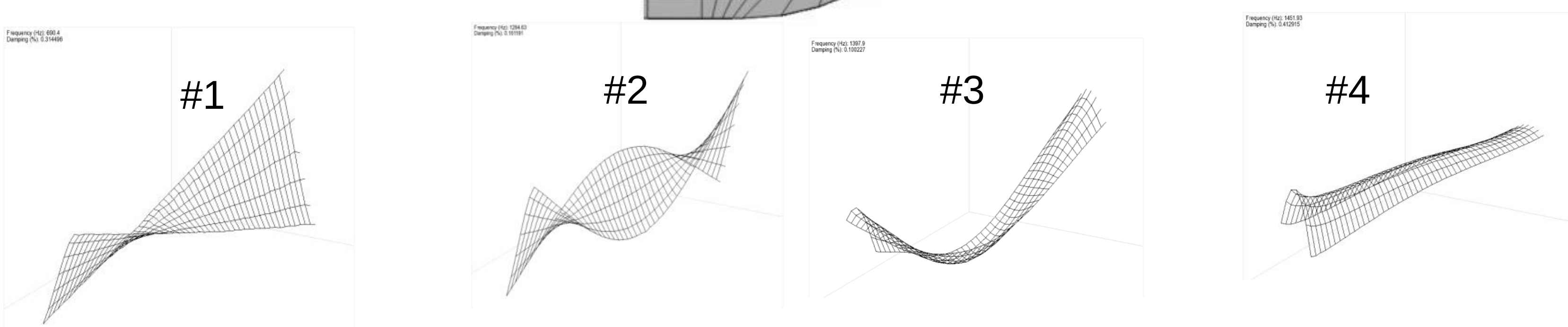
- Effect of geometry – mode shapes of “free” beam

m1

m2



Experiment [Hz]	Model 1 [Hz]	Model 2 [Hz]
690	970	**688**
1285	1503	**1252**
1398-1411	1477	**1338**
1451	1570	1435



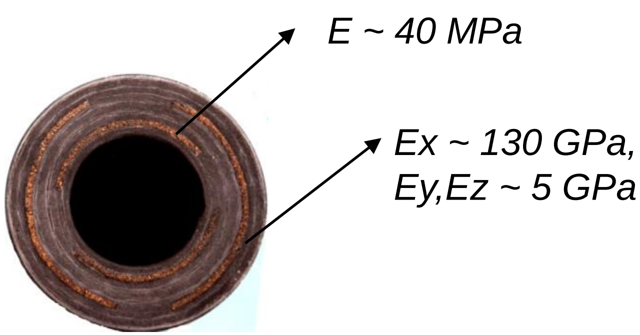
– due to the geometry simplifications, shell model or even continuum shell model with 1 element per thickness not working for the mode shapes and frequency prediction except the bending mode

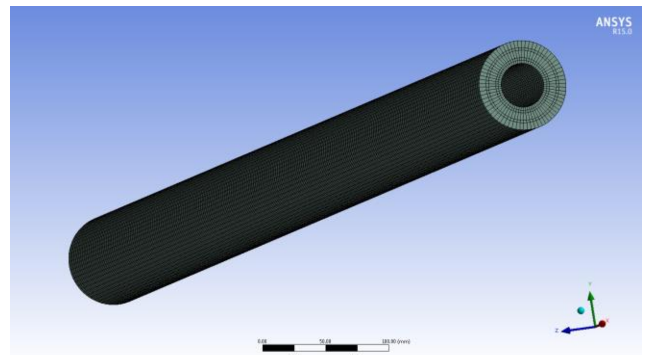
– more detailed geometry from continuum shells in good relation with experiment

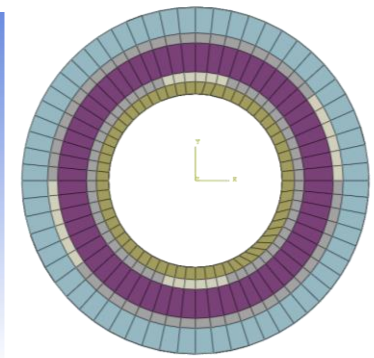
– both models work for the bending modes in a similar way

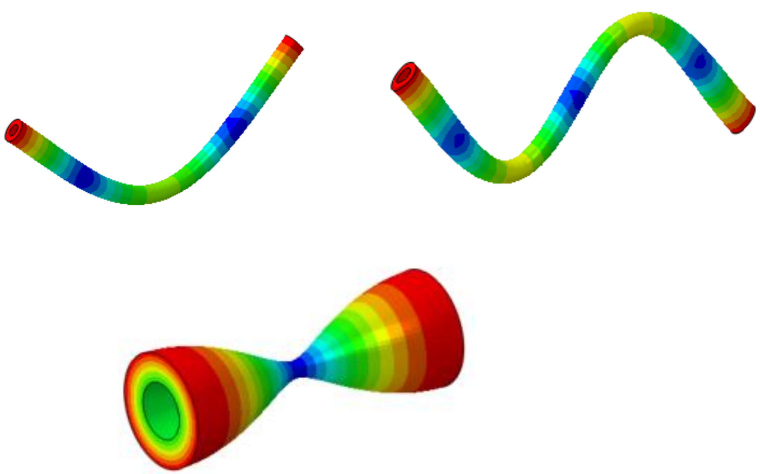
Example – beam profile coupons

- Effect of element selection
  - important for hybrid composites with damping layers that have significantly higher compliance
  - separation of elements for damping material necessary





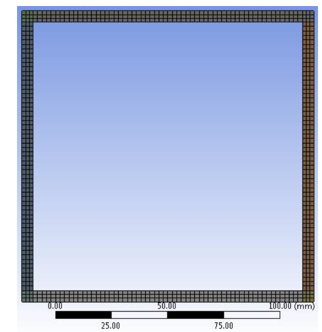


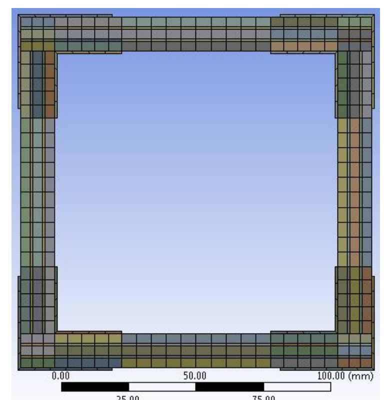


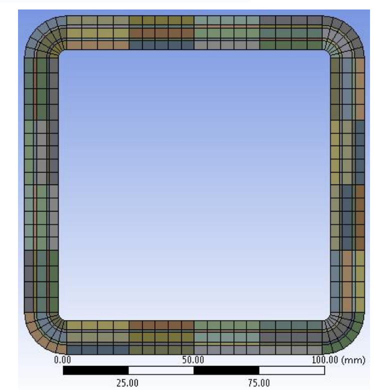
	m01	m02	m03
Experiment [Hz]	**452**	**1123**	***1841***
FEA – solid shells [Hz]	484	1196	*1298*
FEA – one shell [Hz]	***344***	***669***	***590***
MFEA – solid shells, mat hom [Hz]	495	1223	*1231*

Example – beam profile coupons

- Spindle ram coupons
  - comparison of steel, cast iron, CFRP plates assembly and profile by winding
  - FE models
    - derived from the previous cases
    - separation of elements for damping layers





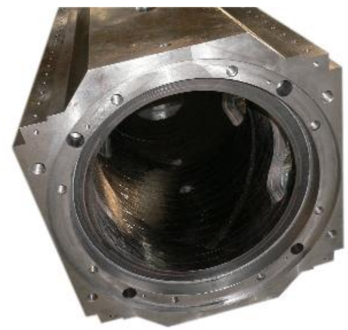


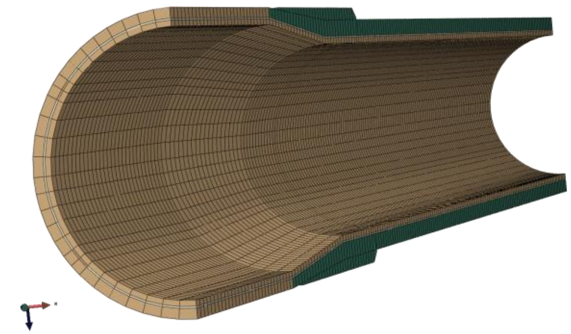
	Cast iron	Welded steel	CFRP plates	Filament winding
FEA_1 [Hz]	457	585	905	1078
Exp_1 [Hz]	493	582	822	1028
FEA_2 [Hz]		585	911	1078
Exp_2 [Hz]		587	841	1035

- Experiment to FEA deviation in bending bellow 10%

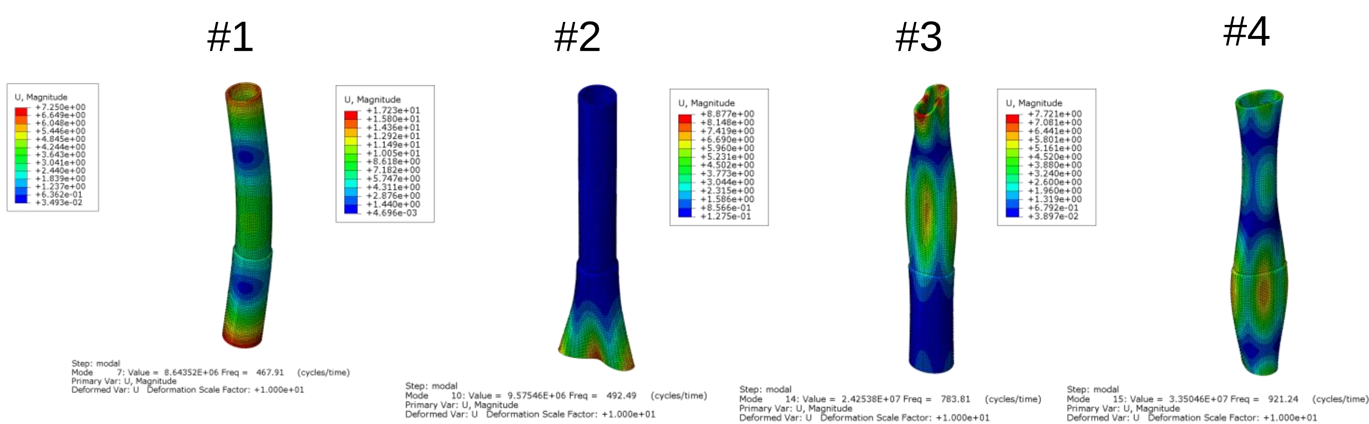
Example - hybrid spindle ram

- Modelling of hybrid spindle ram and its composite reinforcement
  - Combination of carbon/epoxy layers from PITCH and PAN fibres, 1 integrated damping layer
  - Solid shell model with element stacking
  - For bending modes deviation between FEA and experiment bellow 5%
  - For other modes deviation up to 20% and more



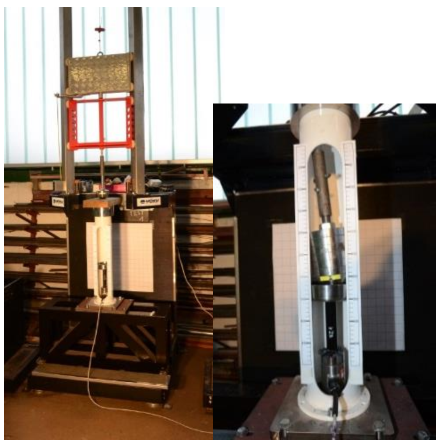


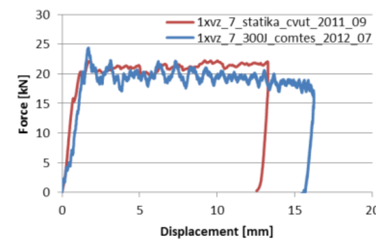
Mode [-]	f_{EXP} [Hz]	f_{FEA} [Hz]	$\Delta f_{FEA/EXP}$ [%]	
1	492	468	-4,9	1st bending
2	493	596	20,9	
3	784	715	-8,8	
4	922	921	-0,1	
5	1 158	1 124	-2,9	2nd bending

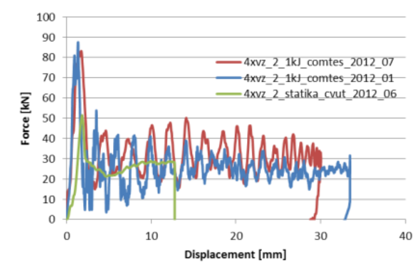


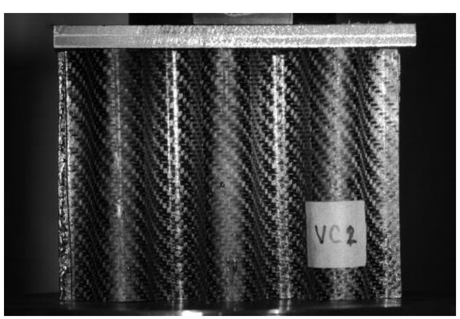
Example – material degradation

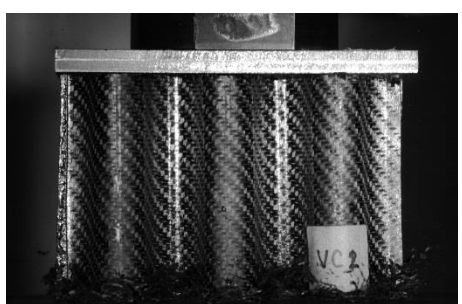
- Crash absorbers simulation
  - ability of progressive damaging of fibre composites to transform kinetic energy into the deformation energy in the safety element ii v bezpečnostním členu

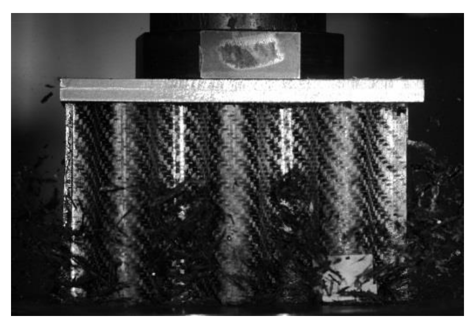












Example – material degradation

- Simulations of progressive damaging
  - progressive damage implemented by failure criteria (Chang-Chang)
    - element stiffness degradation in respect to achieving criterion
    - after the set level of degradation – element removal
  - Chang-Chang failure criterion
    - fibre failure in tension

stiffness change of element for $f_{ft}=1$

$$f_{ft} = \left(\frac{\hat{\sigma}_{11}}{X^T}\right)^2 + \beta\left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2, \quad where\ 0 \le \beta \le 1,$$

$$E_{11} = E_{22} = G_{12} = \nu_{12} = \nu_{21} = 0$$

- fibre failure in compression

stiffness change of element for $f_{fc}=1$

$$f_{fc} = \left(\frac{\hat{\sigma}_{11}}{X^C}\right)^2,$$

$$E_{11} = \nu_{12} = \nu_{21} = 0,$$

- matrix failure in tension

stiffness change of element for $f_{mt}=1$

$$f_{mt} = \left(\frac{\hat{\sigma}_{22}}{Y^T}\right)^2 + \left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2,$$

$$E_{22} = G_{12} = \nu_{21} = 0,$$

- matrix failure in compression

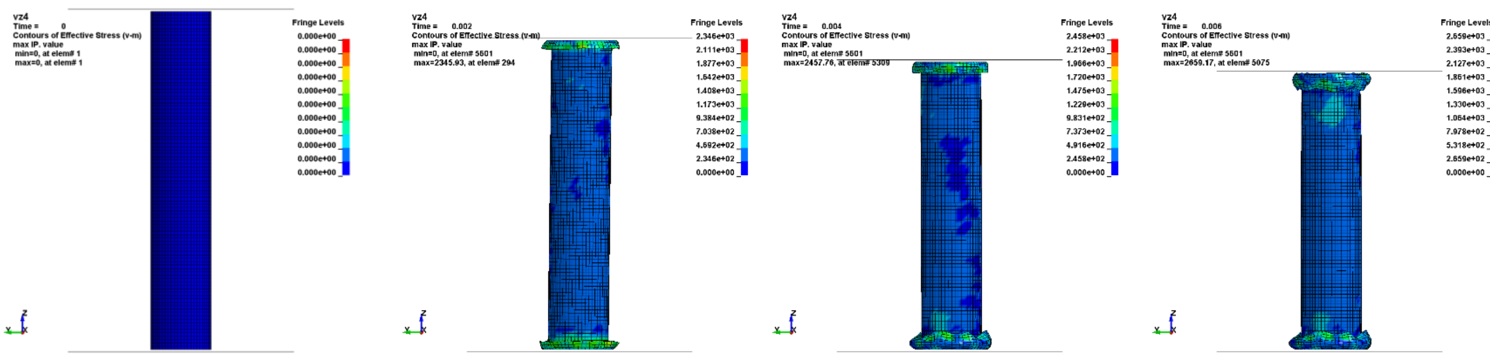
stiffness change of element for $f_{mc}=1$

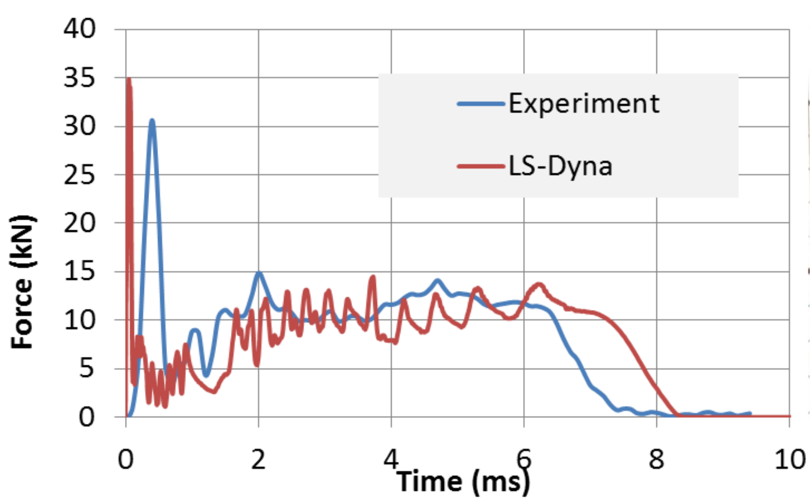
$$f_{mc} = \left(\frac{\hat{\sigma}_{22}}{2S^T}\right)^2 + \left[\left(\frac{Y^C}{2S^T}\right)^2 - 1\right]\frac{\hat{\sigma}_{22}}{Y^C} + \left(\frac{\hat{\sigma}_{12}}{S^L}\right)^2.$$

$$E_{22} = G_{12} = \nu_{12} = \nu_{21} = 0$$

Example – material degradation

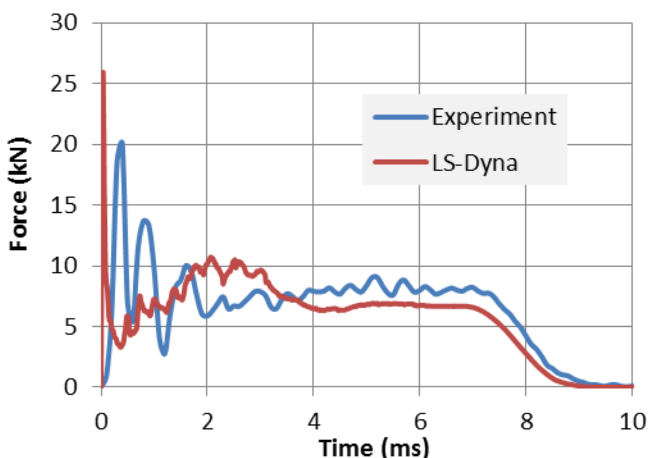
- Simulations of progressive damaging
  - LS-Dyna: shell element with 1 element per the coupon thickness
  - good match with experimental behaviour

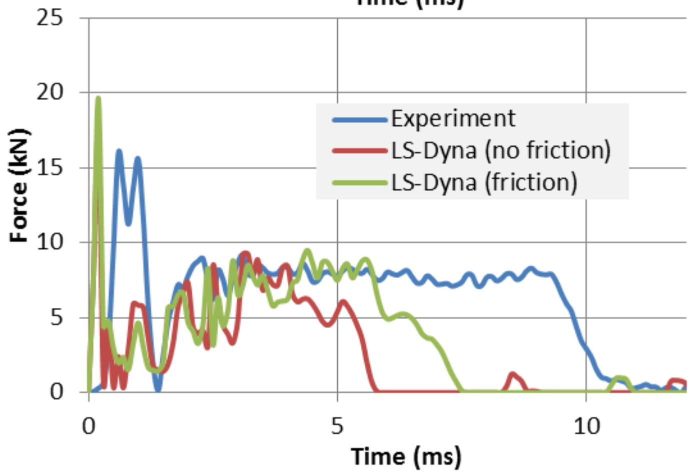




Example – material degradation

- Simulations of progressive damaging

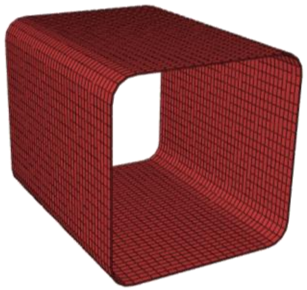




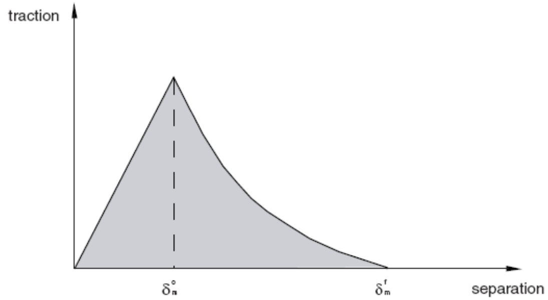
Example – adhesive joints of components

- Simulation of adhesive joint failure in composite shafts with bonded metal endings
  - prediction of the joint degradation – cohesive elements
    - damage initiation
    - damage growth
    - after the determined degradation element removal
  - demonstration – from the development of composite shafts for the machine tool industry

Adhesive joint model



Damage initiation and growth



Example – adhesive joints of components

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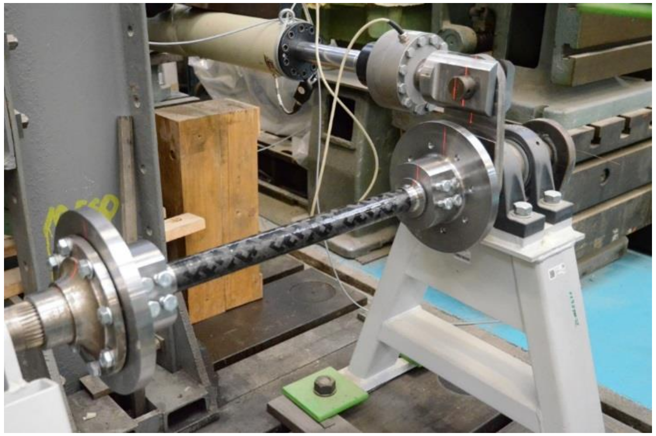
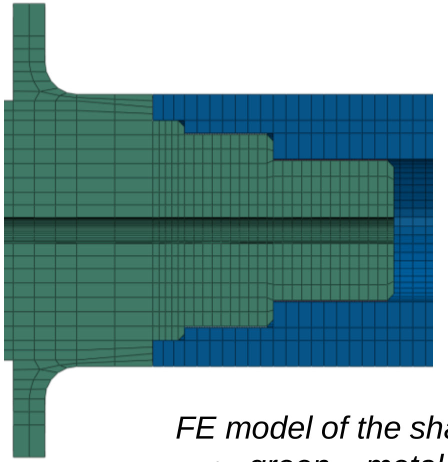
Experimental testing – loading of shafts in torsion

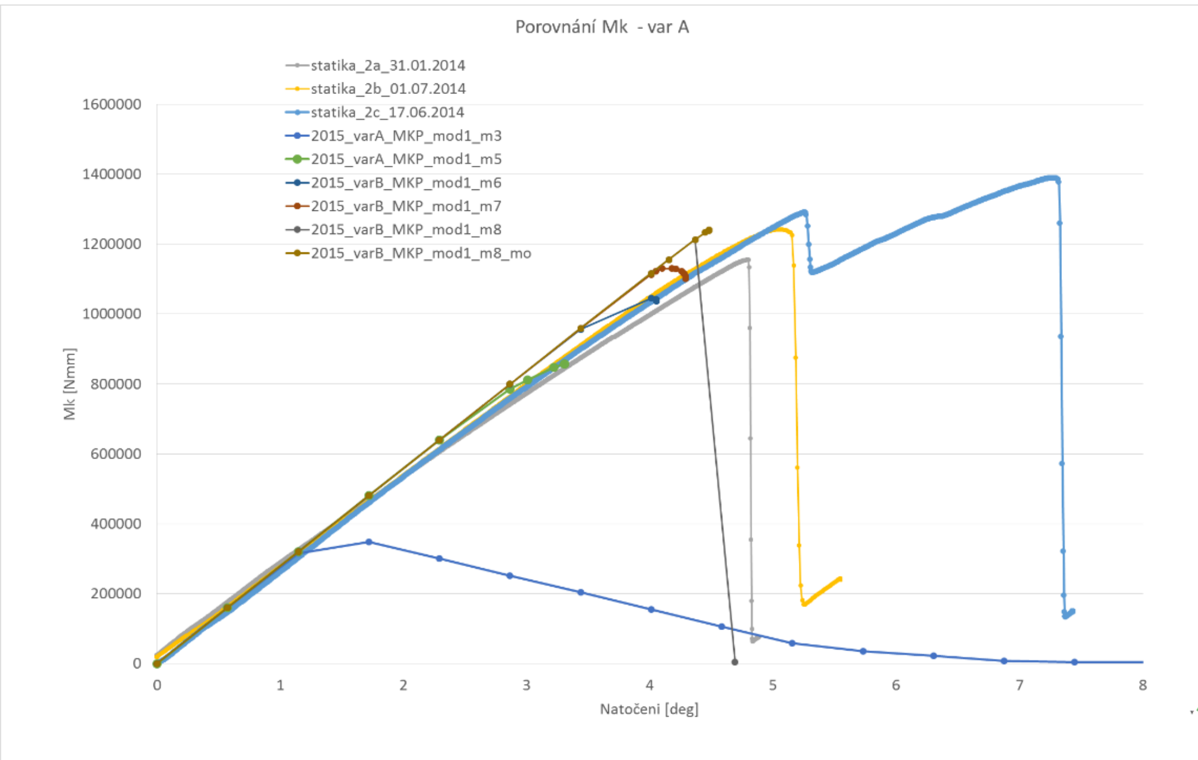


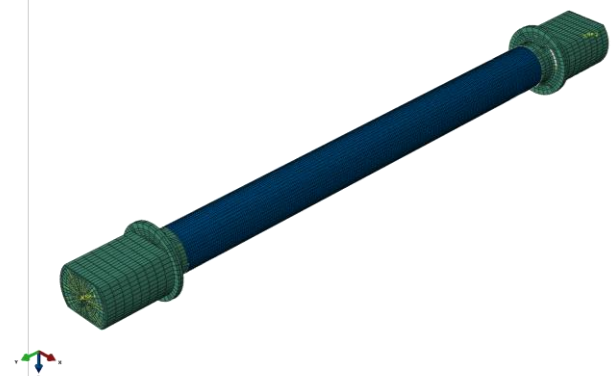


FE model of the shaft ending
- *green – metal ending*
- *blue – composite shaft*

Example – adhesive joints of components

- Simulation of adhesive joint failure in composite shafts with bonded metal endings
  - Finite element simulation in comparison with experimental behaviour
    - Comparison of reaction moment and rotation





- acceptable prediction of maximal loading moment

Example – sandwich panels

Sandwich structures

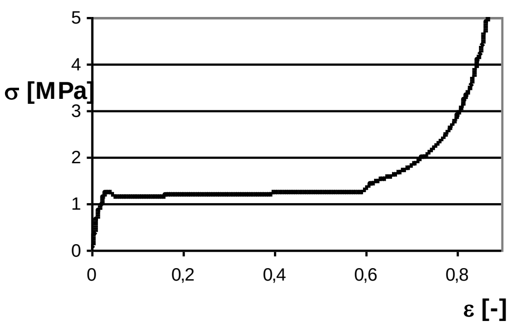
+ low-weight design

+ high bending stiffness

+ high natural frequencies

- low compressive strength

- difficulty when joining



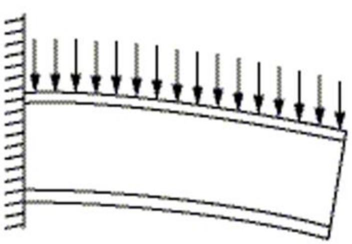
Example – sandwich panels

Necessary to include the effect of transverse shearing

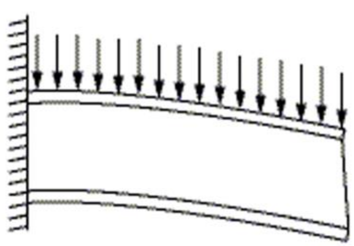
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{26} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{S}_{11} & \widetilde{S}_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{S}_{12} & \widetilde{S}_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}.$$

FEA

- due to transverse shearing, the normal to the reference surface rotates
- shell element cannot behave in this way
  - with some exceptions (sandwich logic, balance of energy)



Deformed shape without sandwich option



Deformed shape with sandwich option

Ansys:

- Shell91 – former element for sandwich simulations
- nowadays Shell181,281 - elements model the transverse-shear deflection using an energy-equivalence method

Example – sandwich panels

Approaches for FE modelling of sandwich panels

Shell elements

- generally care must be taken as the approach of using 1 shell element for the sandwich structure might work only for specified shells in one FE solver, but not in other solver

- problematic behaviour of the core with larger compliance (stiffness is lower by 3 orders in comparison with skins – does not meet the conditions for shells)

Solid elements

- core and skins modelled by solid elements, or solid-shell elements
- might be problematic for composite skins

Combination of solid and shell elements

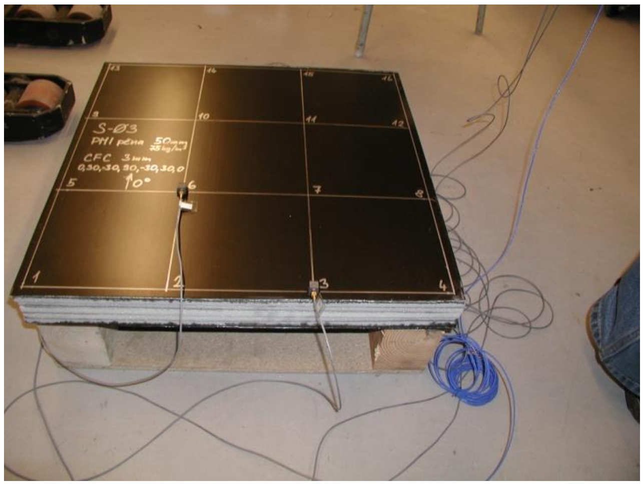
- core modelled by solid elements
- skins modelled by shell or solid shell elements

Example – sandwich panels

2006 – models shell99 skins, solid95 core
Comparison of 3point bending test – deflection of the beam

Skin	Core	Weight [kg]	Mid Span Deflection [mm]	FEA results [mm]
C/E	Roh71 c=30mm	0.40	1.06	1.17
C/E	Roh71 c=50mm	0.46	0.73	0.78
C/E	Roh110 c=30mm	0,45	0.68	0.77
C/E	Roh110 c=50mm	0.52	0.45 0.41*	0.50 0.44*
C/E	Roh110 c=50mm	0.84	0.33 0.30*	0.35 0.32*
C/E	Al250 c=50mm	0.76	0.16-0.20	0.13
Steel	Alporas230 c=50mm	2.60	0.11-0.16 0.09*	0.08 0.07*
Steel	Alporas230 c=30mm	2.44	0.15-0.24 0.12*	0.13 0.12*
Steel	Al250 c=50mm	2.64	0.09-0.13 0.06*	0.08 0.06*
C/E	AL honeycomb core	0,46	0.22	-

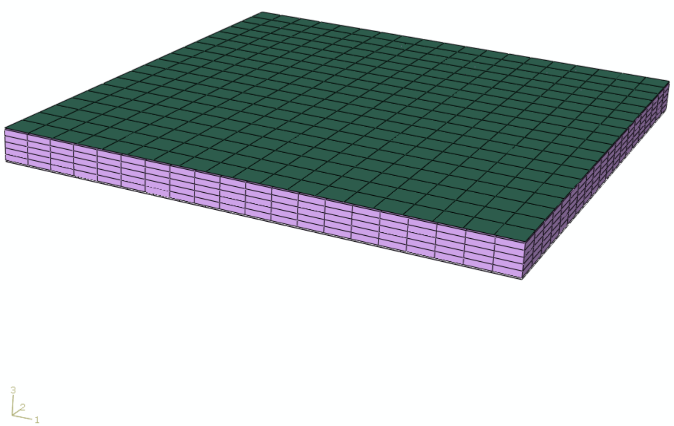
Example – sandwich panels



FE model 1: Ansys
skins: Shell99, 7 layers
core: Solid95
Skins are at the top (bottom) surface of the solid core; with offset from the midsurface
Nodes of the shell skins are shared with the nodes of the solid core surface

FE model 2: Abaqus
skin: S4R
core: C3D8i
*Tie constraint between skin and shell

Experimental modal analysis
Difference between FEM and experiment bellow 15% for the first bending frequency



	3mm C/E, 30mm PMI			3mm C/E, 50mm PMI		
Mód	f_{exp} [Hz]	f_{mkp1} [Hz]	f_{mkp2} [Hz]	f_{exp} [Hz]	f_{mkp1} [Hz]	f_{mkp2} [Hz]
1	415.7	373.4	376.4	529.2	469.9	475.3
2	539.8	539.8	543.9	747.9	675.4	683
3	713.5	618.7	624.7	851.6	739.7	749.2
4	764	660.1	668.5	924.5	799.7	812.1

Thank you for your attention!

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