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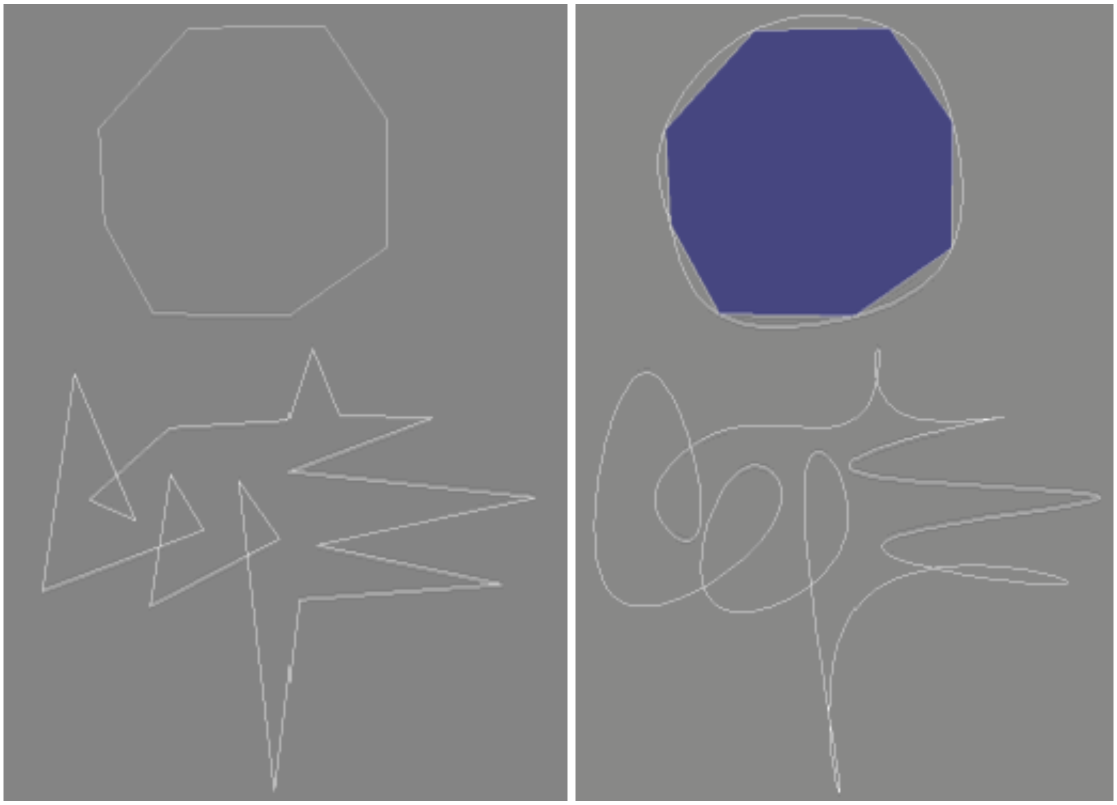
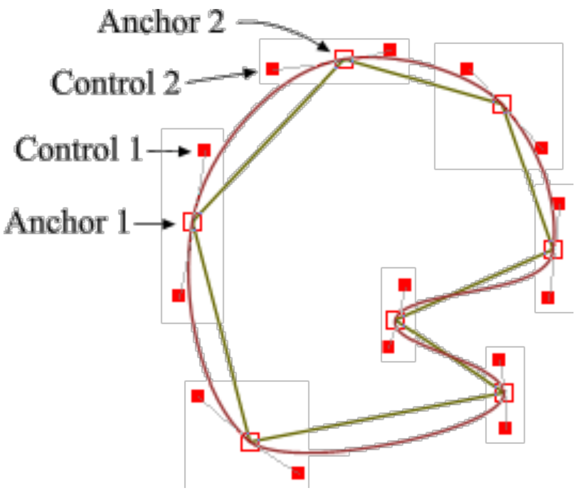
Interpolation with Bezier Curves

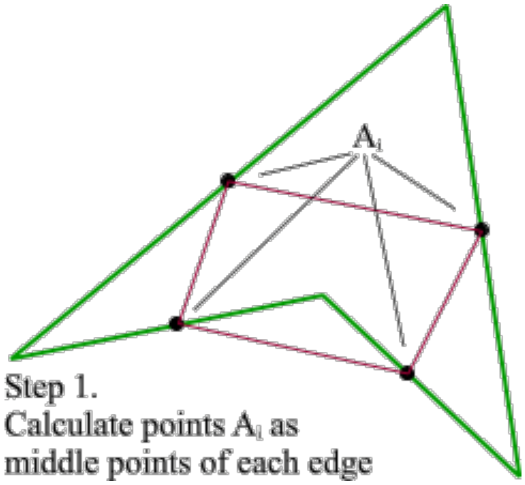
A very simple method of smoothing polygons

Initially, there was a question in comp.graphic.algorithms how to interpolate a polygon with a curve in such a way that the resulting curve would be smooth and hit all its vertices. Gernot Hoffmann suggested to use a well-known B-Spline interpolation. Here is his original article. B-Spline works good and it behaves like an elastic ruler fixed in the polygon vertices.





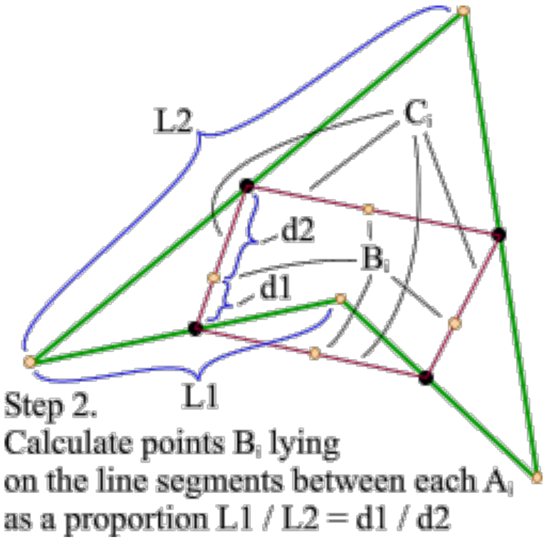
But I had a gut feeling that there must be a simpler method. For example, approximation with cubic Bezier curves. A Bezier curve has two anchor points (begin and end) and two control ones (CP) that determine its shape. More information about Bezier curves can be found using any search engine, for example, on Paul Bourke's excellent site. Our anchor points are given, they are pair of vertices of the polygon. The question was, how to calculate the control points. I ran **Xara X** and drew this picture. It was pretty easy and I decided to try to calculate their coordinates. It was obvious that the control points of two adjacent edges plus the vertex between them should form one straight line. Only in this case the two adjacent curves will be connected smoothly. So, the two CP should be the a reflection of each other, but… not quite. Reflection assumes equal distances from the central point. For our case it's not correct. First, I tried to calculate a bisectrix between two edges and then take points on the perpendicular to it. But as shown in the picture, the CP not always lie on the perpendicular to the bisectrix.



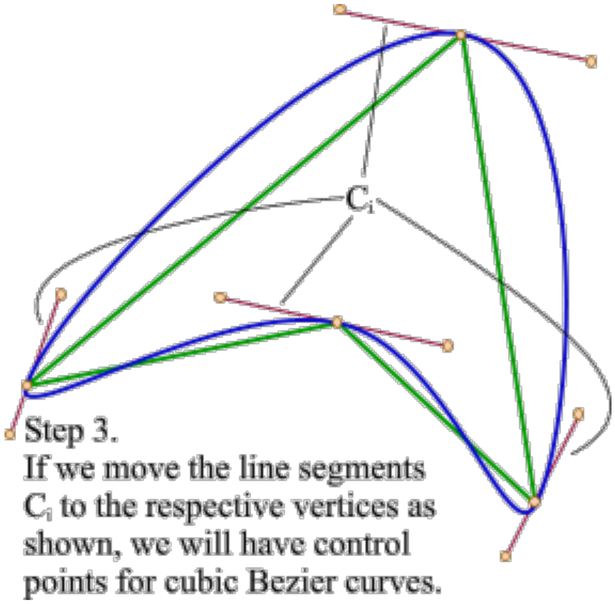
Step 1.
Calculate points A_i as
middle points of each edge

Finally, I found a very simple method that does not require any complicated math. First, we take the polygon and calculate the middle points A_i of its edges.

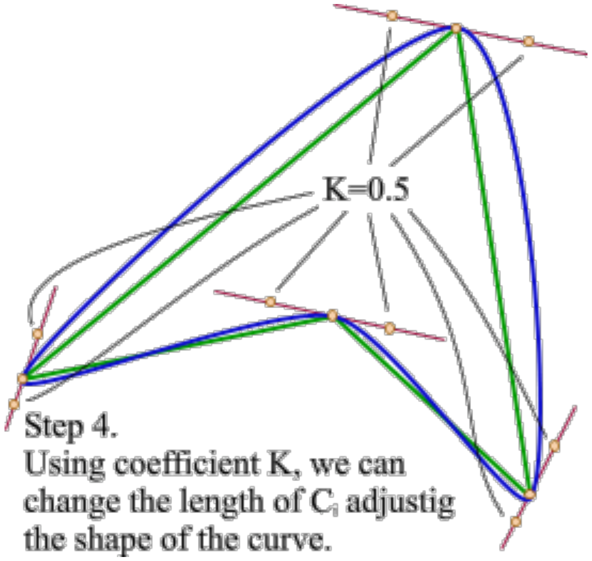
Here we have line segments C_i that connect two points A_i of the adjacent segments. Then, we should calculate points B_i as shown in this picture.



Step 2.
Calculate points B_i lying
on the line segments between each A_i
as a proportion $L1 / L2 = d1 / d2$

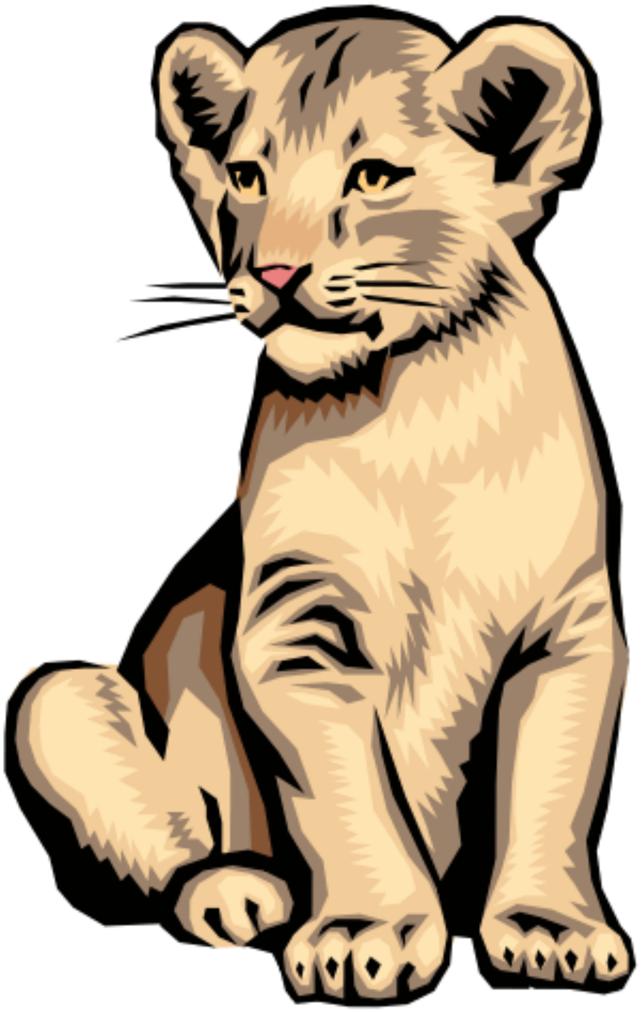

C_i
Step 3.
If we move the line segments
C_i to the respective vertices as
shown, we will have control
points for cubic Bezier curves.

The third step is final. We simply move the line segments C_i in such a way that their points B_i coincide with the respective vertices. That's it, we calculated the control points for our Bezier curve and the result looks good.

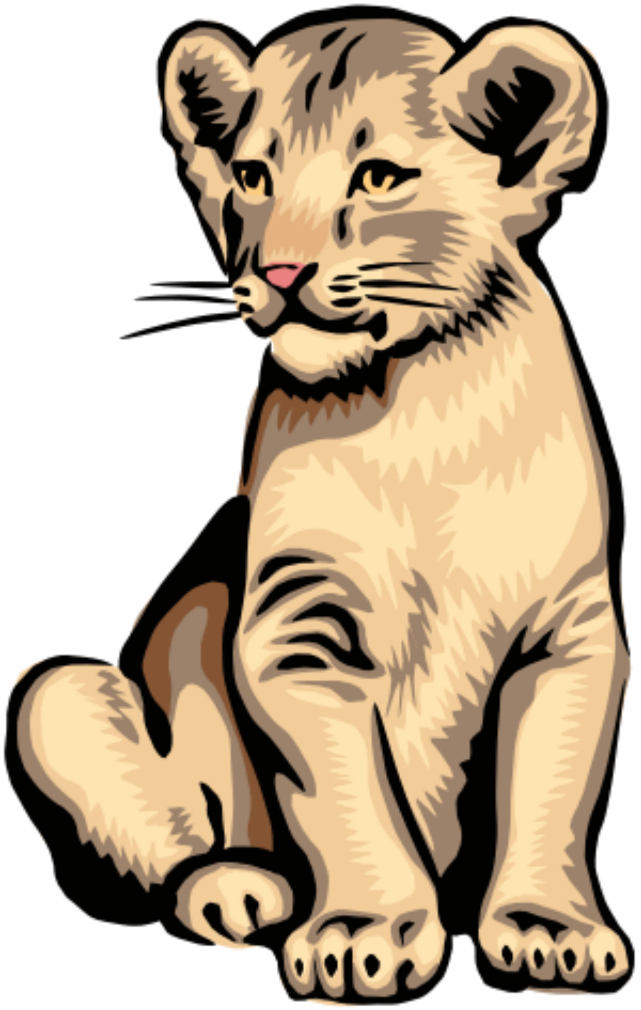

K=0.5
Step 4.
Using coefficient K, we can
change the length of C_i adjustig
the shape of the curve.

One little improvement. Since we have a straight line that determines the place of our control points, we can move them as we want, changing the shape of the resulting curve. I used a simple coefficient K that moves the points along the line relatively to the initial distance between vertices and control points. The closer the control points to the vertices are the sharper figure will be obtained.

Below is the result of rendering a popular in **SVG** lion in its original form and with Bezier interpolation with K=1.0

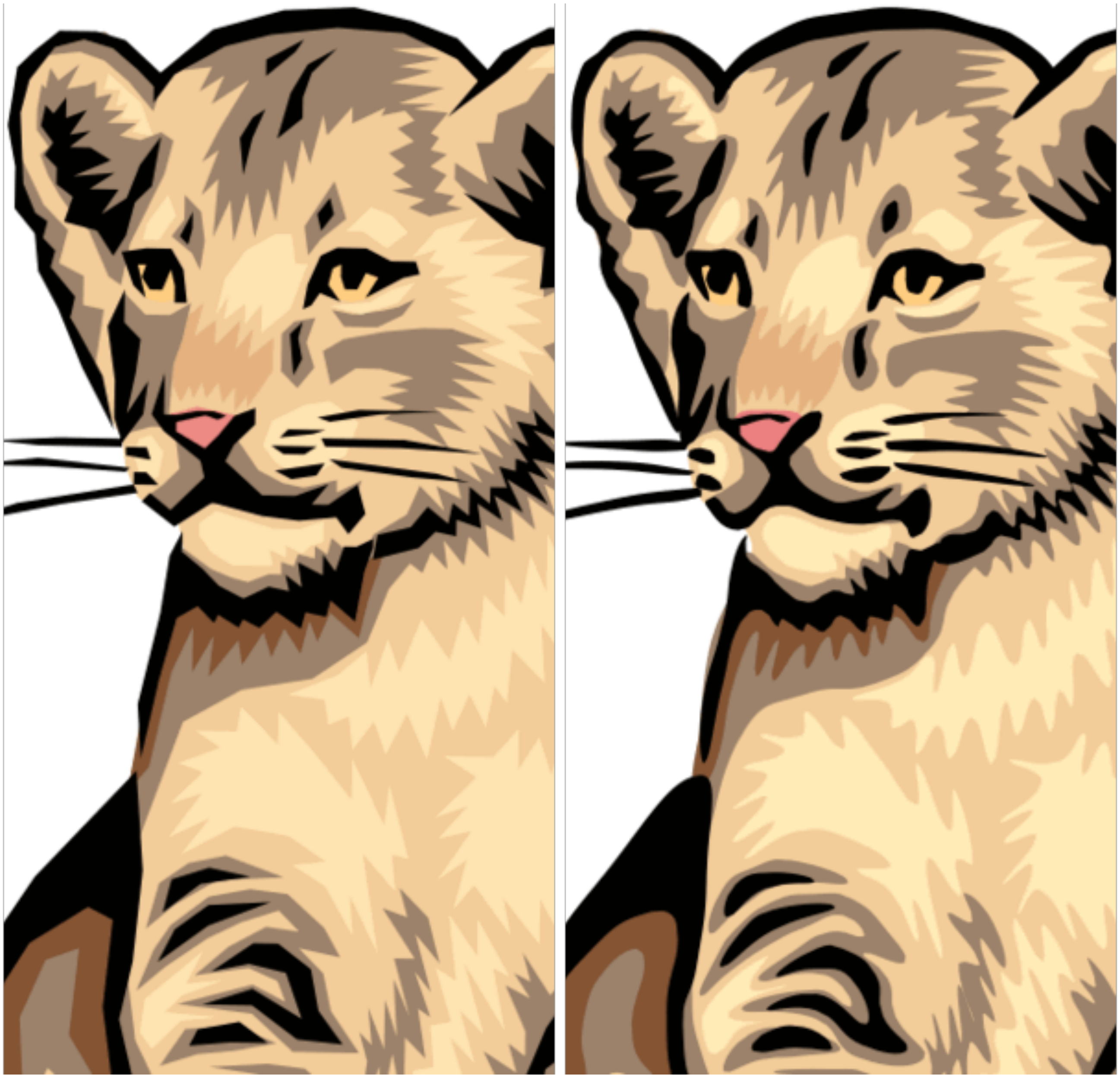


Original polygons

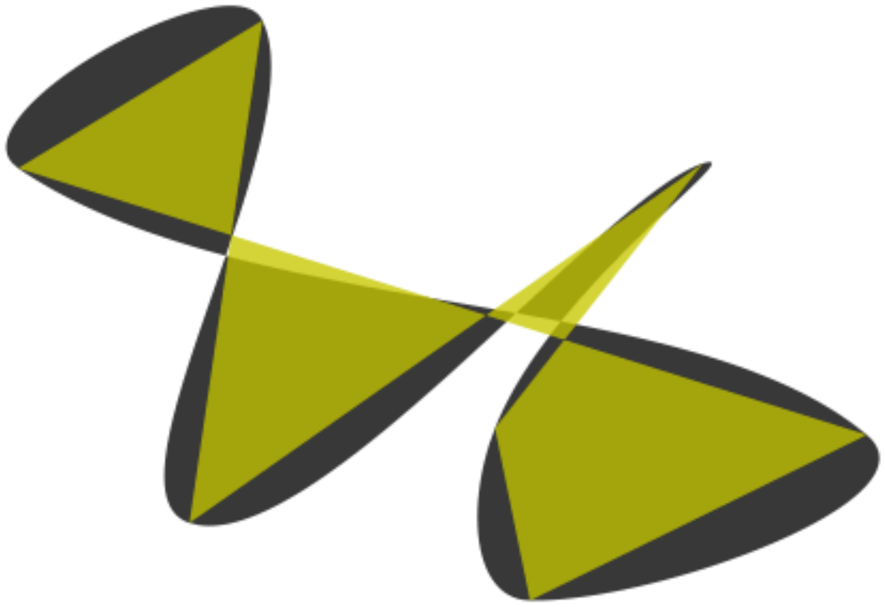


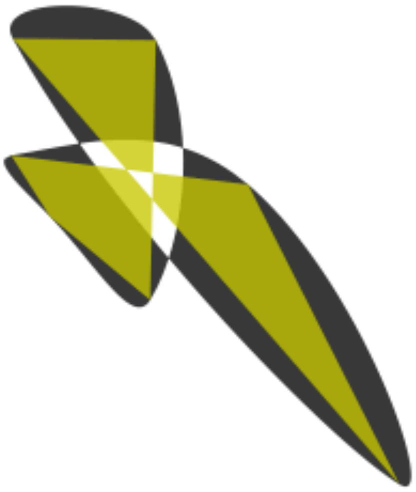
Smooth polygons

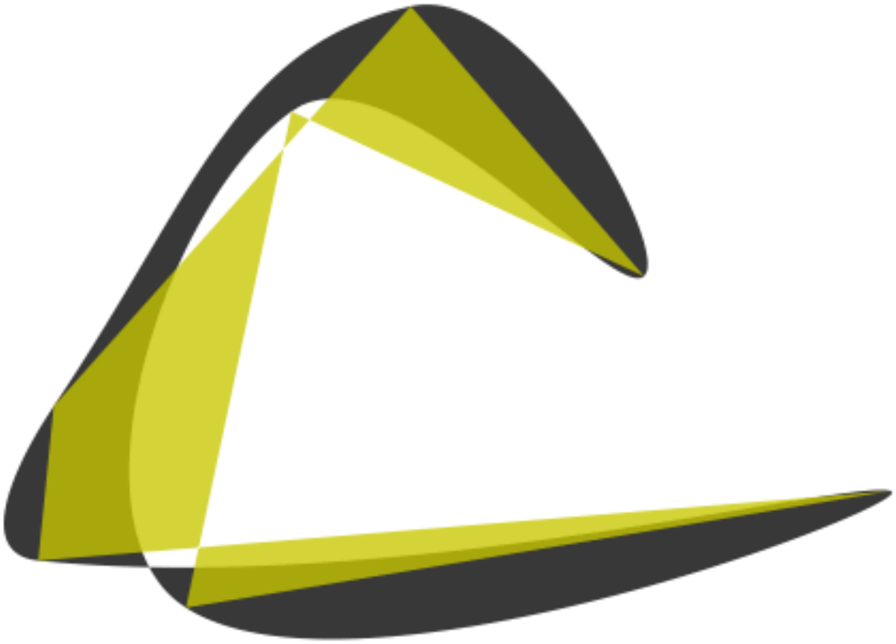
And the enlarged ones.



The method works quite well with self-intersecting polygons. The examples below show that the result is pretty interesting.







This method is pure heuristic and empiric. It probably gives a wrong result from the point of view of strict mathematical modeling. But in practice the result is good enough and it requires absolute minimum of calculations. Below is the source code that has been used to generate the lions shown above. It's not optimal and just an illustration. It calculates some variables twice, while in real programs we can store and reuse them in the consecutive steps.

```
    // Assume we need to calculate the control
    // points between (x1,y1) and (x2,y2).
    // Then x0,y0 - the previous vertex,
    //      x3,y3 - the next one.

    double xc1 = (x0 + x1) / 2.0;
    double yc1 = (y0 + y1) / 2.0;
    double xc2 = (x1 + x2) / 2.0;
    double yc2 = (y1 + y2) / 2.0;
    double xc3 = (x2 + x3) / 2.0;
    double yc3 = (y2 + y3) / 2.0;

    double len1 = sqrt((x1-x0) * (x1-x0) + (y1-y0) * (y1-y0));
    double len2 = sqrt((x2-x1) * (x2-x1) + (y2-y1) * (y2-y1));
    double len3 = sqrt((x3-x2) * (x3-x2) + (y3-y2) * (y3-y2));

    double k1 = len1 / (len1 + len2);
    double k2 = len2 / (len2 + len3);

    double xm1 = xc1 + (xc2 - xc1) * k1;
    double ym1 = yc1 + (yc2 - yc1) * k1;

    double xm2 = xc2 + (xc3 - xc2) * k2;
    double ym2 = yc2 + (yc3 - yc2) * k2;

    // Resulting control points. Here smooth_value is mentioned
    // above coefficient K whose value should be in range [0...1].
    ctrl1_x = xm1 + (xc2 - xm1) * smooth_value + x1 - xm1;
    ctrl1_y = ym1 + (yc2 - ym1) * smooth_value + y1 - ym1;
    ctrl2_x = xm2 + (xc2 - xm2) * smooth_value + x2 - xm2;
    ctrl2_y = ym2 + (yc2 - ym2) * smooth_value + y2 - ym2;
```

And the source code of an approximation with a cubic Bezier curve.

```
// Number of intermediate points between two source ones,
// Actually, this value should be calculated in some way,
// Obviously, depending on the real length of the curve.
// But I don't know any elegant and fast solution for this
// problem.
#define NUM_STEPS 20

void curve4(Polygon* p,
            double x1, double y1,   //Anchor1
            double x2, double y2,   //Control1
            double x3, double y3,   //Control2
            double x4, double y4)   //Anchor2
{
    double dx1 = x2 - x1;
    double dy1 = y2 - y1;
    double dx2 = x3 - x2;
    double dy2 = y3 - y2;
    double dx3 = x4 - x3;
    double dy3 = y4 - y3;

    double subdiv_step  = 1.0 / (NUM_STEPS + 1);
    double subdiv_step2 = subdiv_step*subdiv_step;
    double subdiv_step3 = subdiv_step*subdiv_step*subdiv_step;

    double pre1 = 3.0 * subdiv_step;
    double pre2 = 3.0 * subdiv_step2;
    double pre4 = 6.0 * subdiv_step2;
    double pre5 = 6.0 * subdiv_step3;

    double tmp1x = x1 - x2 * 2.0 + x3;
    double tmp1y = y1 - y2 * 2.0 + y3;

    double tmp2x = (x2 - x3)*3.0 - x1 + x4;
    double tmp2y = (y2 - y3)*3.0 - y1 + y4;

    double fx = x1;
    double fy = y1;

    double dfx = (x2 - x1)*pre1 + tmp1x*pre2 + tmp2x*subdiv_step3;
    double dfy = (y2 - y1)*pre1 + tmp1y*pre2 + tmp2y*subdiv_step3;

    double ddfx = tmp1x*pre4 + tmp2x*pre5;
    double ddfy = tmp1y*pre4 + tmp2y*pre5;

    double dddfx = tmp2x*pre5;
    double dddfy = tmp2y*pre5;

    int step = NUM_STEPS;

    // Suppose, we have some abstract object Polygon which
    // has method AddVertex(x, y), similar to LineTo in
    // many graphical APIs.
    // Note, that the loop has only operation add!
    while(step--)
    {
        fx   += dfx;
        fy   += dfy;
        dfx  += ddfx;
        dfy  += ddfy;
        ddfx += dddfx;
        ddfy += dddfy;
        p->AddVertex(fx, fy);
    }
    p->AddVertex(x4, y4); // Last step must go exactly to x4, y4
}
```

You can download a working application for Windows that renders the lion, rotates and scales it, and generates random polygons. Interpolation with Bezier curves (bezier_interpolation.zip). Press left mouse button and drag to rotate and scale the image around the center point. Press right mouse button and drag left-right to change the coefficient of smoothing (K). Value K=1 is about 100 pixels from the left border of the window. Each left double-click generates a random polygon. You can also rotate and scale it, and change K.