Title:	MAST-like merging flux in Cartesian geometry
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This work aims to reproduce simulations of merging current filaments (flux ropes) in a MAST-like Cartesian geometry. Parameters are taken from [Stanier 2013]: A.Stanier et al. Phys. Plasmas 20, 122302 (2013); doi: 10.1063/1.4830104.

All source code, inputs files, and analysis scripts used here are publicly available at `https://github.com/boutproject/merging-filaments`

Model

A 2D zero-β model in Cartesian geometry. The vorticity ω and electromagnetic potential $A_{||}$ are evolved with a constant density n_0 and temperature $T_e = T_i$. The magnetic field consists of a constant "toroidal" field B_0, and a time-evolving "poloidal" field so that the total field is:

$$\mathbf{B} = B_0 \mathbf{e}_\phi - \mathbf{e}_\phi \times \nabla A_{||} \tag{1}$$

so $A_{||}$ is the poloidal flux. The equations in SI units are:

$$\frac{\partial \omega}{\partial t} + \frac{1}{B_0}\mathbf{b}_0 \times \nabla\phi \cdot \nabla\omega = \nabla \cdot \left(\mathbf{b} J_{||}\right) + \nu \nabla_\perp^2 \omega \tag{2a}$$

$$\frac{\partial A_{||}}{\partial t} = -\mathbf{b}\cdot\nabla\phi + \eta J_{||} \tag{2b}$$

$$\omega = \nabla \cdot \left(\frac{\rho_0}{B_0^2}\nabla_\perp \phi\right) \tag{2c}$$

$$J_{||} = -\frac{1}{\mu_0}\nabla_\perp^2 A_{||} \tag{2d}$$

Where $\mathbf{b}_0 = \mathbf{e}_\phi$ is the "toroidal" magnetic field unit vector, and $\mathbf{b} = \mathbf{B}/B_0$ is the unit vector along the total magnetic field, assuming the poloidal magnetic field is small compared to the toroidal field. $\nabla_\perp = \nabla - \mathbf{b}_0\mathbf{b}_0 \cdot \nabla$ is the component of the gradient in the poloidal plane.

The dissipation terms are the kinematic viscosity ν (units m^2/s) and resistivity η (units Ωm).

Normalised equations

Normalising to a reference mass density ρ_0 gives an Alfvén timescale

$$\tau_A = \sqrt{\mu_0 \rho_0} \tag{3}$$

where we take a reference magnetic field of 1m and length of 1m. Other quantities are normalised as:

$$\hat{J}_{||} = \mu_0 J_{||} \qquad \hat{A}_{||} = A_{||} \tag{4a}$$

$$\hat{\omega} = \frac{\tau_A}{\rho_0}\omega \qquad \hat{\phi} = \tau_A \phi \tag{4b}$$

$$\hat{\nu} = \tau_A \nu \qquad \hat{\eta} = \frac{\tau_A}{\mu_0}\eta \tag{4c}$$

where quantities with hats on are normalised, and without hats are in SI units. Taking a 2D domain, assuming no variation in the toroidal direction, the normalised equations solved are:

$$\frac{\partial \hat{\omega}}{\partial \hat{t}} + \frac{1}{B_0}\mathbf{b}_0 \times \nabla\hat{\phi}\cdot\nabla\hat{\omega} = -\frac{1}{B_0}\mathbf{b}_0 \times \nabla\hat{A}_{||}\cdot\nabla\hat{J}_{||} + \hat{\nu}\nabla_\perp^2\hat{\omega} \tag{5a}$$

$$\frac{\partial \hat{A}_{||}}{\partial \hat{t}} = \frac{1}{B_0}\mathbf{b}_0 \times \nabla\hat{A}_{||}\cdot\nabla\hat{\phi} + \hat{\eta}\hat{J}_{||} \tag{5b}$$

$$\hat{\omega} = \frac{1}{B_0^2}\nabla_\perp^2\hat{\phi} \tag{5c}$$

$$\hat{J}_{||} = -\nabla_\perp^2\hat{A}_{||} \tag{5d}$$

and all terms of the form $\frac{1}{B_0}\mathbf{b}_0 \times \nabla f \cdot \nabla g$ are implemented as 2^{nd}-order Arakawa brackets.

Simulation inputs

Parameters

Taking the MAST-like plasma parameters from [Stanier 2013], the plasma temperature is taken to be 10eV, and the number density $n_0 = 5 \times 10^{18}\mathrm{m}^{-3}$. The plasma is assumed to be pure Deuterium so $Z = 1, A = 2$. For these parameters the Coulomb logarithm is $\ln\Lambda \simeq 11.6$, and collision times are $\tau_e \simeq 1.9\times 10^{-7}$s and $\tau_i \simeq 1.6 \times 10^{-5}$s. The Spitzer parallel resistivity is therefore $\eta_{S,||} \simeq 1.9 \times 10^{-5}\Omega$m. The Braginskii perpendicular kinematic viscosity is $\nu_{\perp,ci} \simeq 3.9\times 10^{-3}\mathrm{m}^2/\mathrm{s}$, and the gyro-viscosity is $\nu_{\perp,g} \simeq 5\mathrm{m}^2/\mathrm{s}$.

The normalisation timescale here is $\tau_A = 0.145\mu$s, a factor of 2 smaller than in [Stanier 2013], due to the choice of 1T here for normalisation rather than 0.5T. This gives Braginskii normalised dissipation parameters of $\hat{\eta} = 2.2 \times 10^{-6}$ and $\hat{\nu}_\perp = 5.6\times 10^{-10}$ (collisional) or $\hat{\nu}_\perp = 7.2 \times 10^{-7}$ (gyro-viscous).

The normalised values used in [Stanier 2013] are $\eta = 10^{-5}$ and $\nu = 10^{-3}$, which due to the factor of 2 difference in normalisation correspond to $\hat{\eta} = 5\times 10^{-6}$ and $\hat{\nu} = 5 \times 10^{-4}$.

Geometry, initial and boundary conditions

The Cartesian geometry is 1.8m wide (in x), and 4.4m high (in z). Two flux ropes start in the middle of the radial domain ($x = 0.9$m), 0.6m above and below the midplane in z ($z = 2.8$m and $z = 1.6$m). The current profile in each flux rope is

$$J_{||}(r) = \begin{cases} j_m\left(1 - (r/w)^2\right)^2 & \text{if } r \leq w \\ 0 & \text{if } r > w \end{cases} \tag{6}$$

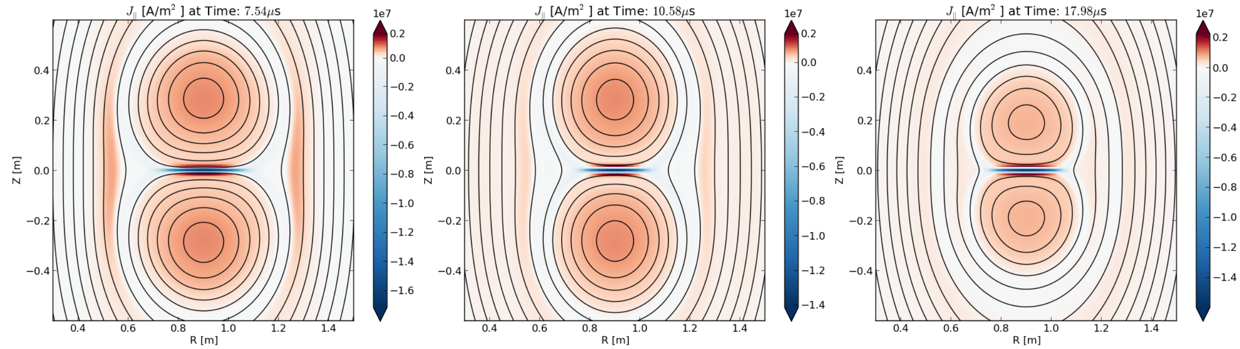

Figure 1: Snapshots of the flux ropes at $t = 7.54\mu$s, 10.58μs and 17.98μs. The current density $J_{||}$ (toroidal current) is shown as the colour, whilst the contour lines are of the poloidal flux $A_{||}$.

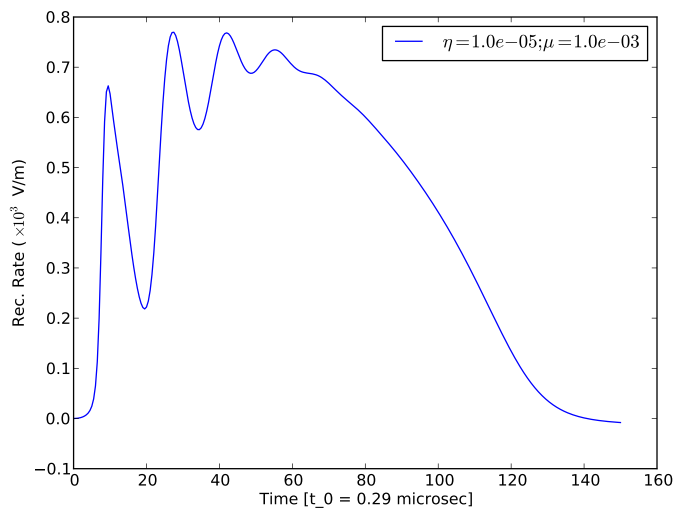

Figure 2: Reconnection rate at the centre of the domain, normalised as in [Stanier 2013]

where r is the radius from the centre of the flux rope, and $w = 0.4$m is the flux rope radius. The initial maximum current density is $j_m = 0.8\text{MA/m}^2$, giving a total plasma current of $I_p = 2 \times (\pi j_m w^2/3) = 268$kA.

The boundary of the domain is assumed superconducting, so we set $\phi = 0$ and $A_{||} = 0$.

1 Results

A uniform 640×1280 grid was used in $x - z$. This is uniform, so the grid spacing in the vertical direction is 3.4mm. In [Stanier 2013] a non-uniform grid was used, with a resolution of 0.23mm in the middle of the domain where the current sheet forms. Snapshots of the current density and flux are shown in figure 1

For a more quantitative comparison with Figure 4 of [Stanier 2013], the reconnection rate (toroidal electric field) at the centre of the domain is shown in figure 2. This is calculated from $\partial_t A_{||} = -E_\phi = -\eta J_{||}$. All quantities in figure 2 have been converted to use the same normalisations as in [Stanier 2013] for direct comparison. The maximum reconnection rate is found to be 769V/m at $t = 7.97\mu$s, compared to 800V/m at $t = 7.99\mu$s in [Stanier 2013]. This value is observed to increase with resolution, so for a 320×640 grid the values are 720V/m at $t = 12.2\mu$s; a 160×320 grid results in 627V/m at 12.3μs.