Title:	Merging filaments in toroidal geometry
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This work aims to reproduce simulations of merging current filaments (flux ropes) in a MAST-like toroidal geometry. Parameters are taken from [Stanier 2013]: A.Stanier et al. Phys. Plasmas 20, 122302 (2013); doi: 10.1063/1.4830104.

All source code, inputs files, and analysis scripts used here are publicly available at `https://github.com/boutproject/merging-filaments`

Model

A 2D zero-β model in toroidal geometry. The vorticity ω and electromagnetic potential $A_{||}$ are evolved with a constant density n_0 and temperature $T_e = T_i$. The magnetic field consists of a constant "toroidal" field B_0, and a time-evolving "poloidal" field so that the total field is:

$$\begin{aligned} \mathbf{B} &= B_0\mathbf{e}_\phi + \nabla\times\left(A_{||}\hat{\mathbf{e}}_\phi\right) && (1) \\ &= B_0\mathbf{e}_\phi + \nabla\psi\times\nabla\phi && (2) \end{aligned}$$

where $\psi = RA_{||}$ is the poloidal flux, and $A_{||}$ is the toroidal component of the vector potential, here approximated by the parallel component. The equations in SI units are:

$$\begin{aligned} \frac{\partial\omega}{\partial t} + \frac{1}{B_0}\mathbf{b}_0\times\nabla\phi\cdot\nabla\omega &= \nabla\cdot\left(\mathbf{b}J_{||}\right) + \nu\nabla_\perp^2\omega && (3a) \\ \frac{\partial A_{||}}{\partial t} &= -\mathbf{b}\cdot\nabla\phi + \eta J_{||} && (3b) \\ \omega &= \nabla\cdot\left(\frac{\rho_0}{B_0^2}\nabla_\perp\phi\right) && (3c) \\ J_{||} &= -\frac{1}{\mu_0}\nabla_\perp^2 A_{||} && (3d) \end{aligned}$$

Where $\mathbf{b}_0 = \mathbf{e}_\phi$ is the "toroidal" magnetic field unit vector, and $\mathbf{b} = \mathbf{B}/B_0$ is the unit vector along the total magnetic field, assuming the poloidal magnetic field is small compared to the toroidal field. $\nabla_\perp = \nabla - \mathbf{b}_0\mathbf{b}_0\cdot\nabla$ is the component of the gradient in the poloidal plane.

The dissipation terms are the kinematic viscosity ν (units m^2/s) and resistivity η (units $\Omega\mathrm{m}$).

Normalised equations

Normalising to a reference mass density ρ_0 gives an Alfvén timescale

$$\tau_A = \sqrt{\mu_0\rho_0} \qquad (4)$$

where we take a reference magnetic field of 1m and length of 1m. Other quantities are normalised as:

$$\hat{J}_{||} = \mu_0 J_{||} \qquad \hat{A}_{||} = A_{||} \qquad \hat{\psi} = \psi \tag{5a}$$

$$\hat{\omega} = \frac{\tau_A}{\rho_0}\omega \qquad \hat{\phi} = \tau_A \phi \tag{5b}$$

$$\hat{\nu} = \tau_A \nu \qquad \hat{\eta} = \frac{\tau_A}{\mu_0}\eta \tag{5c}$$

where quantities with hats on are normalised, and without hats are in SI units. Taking a 2D domain, assuming no variation in the toroidal direction, the normalised equations solved are:

$$\frac{\partial \hat{\omega}}{\partial \hat{t}} + \frac{1}{B_0}\mathbf{b}_0 \times \nabla\hat{\phi}\cdot\nabla\hat{\omega} = -\frac{1}{RB_0}\mathbf{b}_0 \times \nabla\hat{\psi}\cdot\nabla\hat{J}_{||} + \hat{\nu}\nabla_\perp^2\hat{\omega} \tag{6a}$$

$$\frac{\partial \hat{A}_{||}}{\partial \hat{t}} = \frac{1}{RB_0}\mathbf{b}_0 \times \nabla\hat{\psi}\cdot\nabla\hat{\phi} + \hat{\eta}\hat{J}_{||} \tag{6b}$$

$$\hat{\omega} = \frac{1}{B_0^2}\nabla_\perp^2\hat{\phi} \tag{6c}$$

$$\hat{J}_{||} = -\nabla_\perp^2\hat{A}_{||} \tag{6d}$$

and all terms of the form $\frac{1}{B_0}\mathbf{b}_0 \times \nabla f \cdot \nabla g$ are implemented as 2^{nd}-order Arakawa brackets.

Results

The same parameters as for the Cartesian geometry simulations are used, with the same resolution of 640×1280. The differences are the toroidal field, which is $B_\phi = 0.5/R$ [T], and the addition of a uniform vertical field $B_V = -0.03$ [T].

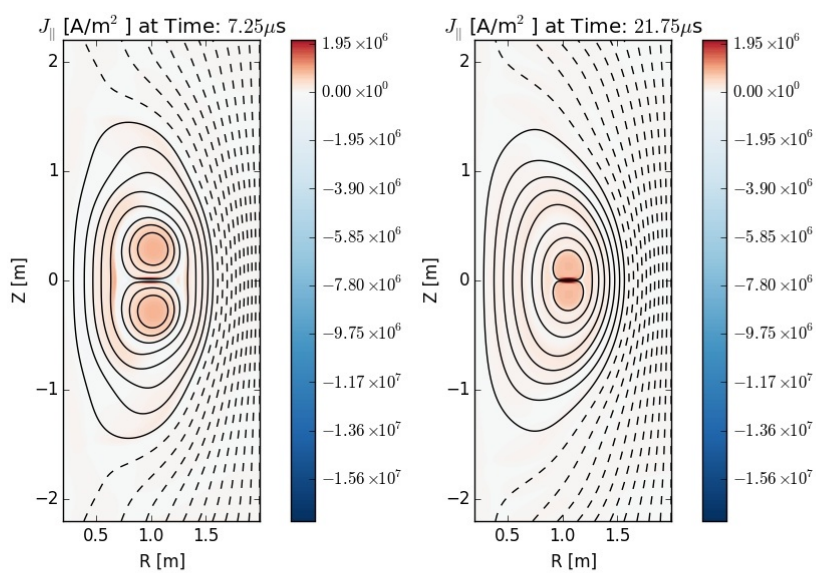

Figure 1: Snapshots of the toroidal current density (colour) and poloidal flux ψ (black lines)