MEMORANDUM

To: file
From: MVU
Subject: Test of electromagnetic resistive drift instability in BOUT
Updated: July 10, 2007

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1 Physics model

For this test problem one needs the following subset of BOUT equations:

- Density

$$\frac{\partial N_i}{\partial t} + \vec{V}_E \cdot \nabla N_i = 0 \tag{1}$$

- Electron parallel momentum

$$m_e \frac{\partial V_{||e}}{\partial t} = -eE_{||} - \frac{1}{N_i}(T_e \partial_{||} N_i) + 0.51\nu_{ei} m_e (V_{||i} - V_{||e}) \tag{2}$$

- Potential vorticity

$$\frac{\partial \varpi}{\partial t} = N_i Z_i e \frac{4\pi V_A^2}{c^2} \nabla_{||} j_{||} \tag{3}$$

where

$$\begin{aligned}
\varpi &= Z_i N_i e \nabla_\perp^2 \phi \\
\vec{V}_E &= c\vec{b}_0 \times \nabla_\perp \phi / B \\
E_{||} &= -\partial_{||}\phi - (1/c)\frac{\partial A_{||}}{\partial t} \\
\nabla_\perp^2 A_{||} &= -(4\pi/c) j_{||} \\
\partial_{||} &= \partial_{||}^0 + \tilde{\vec{b}} \cdot \nabla
\end{aligned} \tag{4}$$

2 Effect on $E_{||}$

First consider the case when magnetic perturbation enters only $E_{||}$. In linearized Fourier-decomposed form these equations lead to a system

$$-i\omega\tilde{N} + \frac{ick_\perp}{B}\tilde{\phi}\frac{N_0}{L_N} = 0 \tag{5}$$

$$N_0 eik_\perp^2\omega\tilde{\phi} = -N_0 e\frac{4\pi V_A^2}{c^2}N_0 eik_{||}\tilde{V}_{||e} \tag{6}$$

$$-i\omega\tilde{V}_{||e} = -\frac{e}{m_e}(-ik_{||}\tilde{\phi} + \frac{i\omega}{c}\tilde{A}_{||}) - \frac{T_{e0}}{N_0 m_e}ik_{||}\tilde{N} - 0.51\nu_{ei}\tilde{V}_{||e} \tag{7}$$

$$-k_\perp^2\tilde{A}_{||} = \frac{4\pi}{c}N_0 e\tilde{V}_{||e} \tag{8}$$

Magnetic perturbation and electron inertia modify the effective collision frequency:

$$0.51\nu_{ei} \to 0.51\nu_{ei} - i\omega - i\omega[\omega_{pe}/ck_\perp]^2 \tag{9}$$

Then the dispersion relation is

$$(\omega - \omega_*)i\sigma_{||} + \omega^2\left(1 - \frac{i\omega}{0.51\nu_{ei}} - \frac{1}{\mu}\frac{i\omega}{0.51\nu_{ei}}\right) = 0 \tag{10}$$

where

$$\mu = (ck_\perp/\omega_{pe})^2 \tag{11}$$

$$\omega_* = k_\perp v_{pe} = k_\perp\frac{V_{te}^2}{\omega_{ce}L_N} \tag{12}$$

$$\sigma_{||} = \left(\frac{k_{||}}{k_\perp}\right)^2\frac{\Omega_{ci}\omega_{ce}}{0.51\nu_{ei}} \tag{13}$$

$$\sigma_\perp = 0.51\nu_{ei}\mu \tag{14}$$

Normalizing all by ω_* we get non-dimensional form

$$(\hat{\omega} - 1)i\hat{\sigma}_{||} + \hat{\omega}^2\left(1 - \mu\frac{i\hat{\omega}}{\hat{\sigma}_\perp} - \frac{i\hat{\omega}}{\hat{\sigma}_\perp}\right) = 0 \tag{15}$$

3 Effect on $\partial_{||}$

The term

$$\frac{1}{N_i m_e}(T_e \partial_{||} N_i) \tag{16}$$

produces two linear terms

$$\frac{1}{N_{i0} m_e}(T_{e0} \partial_{||0} \tilde{N}_i) + \frac{1}{N_{i0} m_e}(T_{e0} \partial_{||1} N_{i0}) \tag{17}$$

The first one has been taken into account, now focus on the second one.

$$\partial_{||1} = \tilde{b} \cdot \nabla = \frac{\nabla A_{||} \times \vec{b}_0}{B} \cdot \nabla \tag{18}$$

Keeping only the leading term one has

$$\tilde{\vec{b}} \cdot \vec{\nabla} N_0 = (\frac{\partial A_{||}}{\partial x}\frac{\partial N_0}{\partial z} - \frac{\partial A_{||}}{\partial z}\frac{\partial N_0}{\partial x}) \to -\frac{\partial A_{||}}{\partial z}\frac{\partial N_0}{\partial x} \to -ik_\perp A_{||}\frac{N_0}{BL_N} \tag{19}$$

In linearized Fourier-decomposed form this leads to a system

$$-i\omega\tilde{N} + \frac{ick_\perp}{B}\tilde{\phi}\frac{N_0}{L_N} = 0 \tag{20}$$

$$N_0 e i k_\perp^2 \omega \tilde{\phi} = -N_0 e \frac{4\pi V_A^2}{c^2} N_0 e i k_{||} \tilde{V}_{||e} \tag{21}$$

$$-i\omega\tilde{V}_{||e} = -\frac{e}{m_e}(-ik_{||}\tilde{\phi}) - \frac{T_{e0}}{N_0 m_e} i k_{||}\tilde{N} + \frac{T_{e0}}{N_0 m_e} i k_\perp A_{||}\frac{N_0}{BL_N} - 0.51\nu_{ei}\tilde{V}_{||e} \tag{22}$$

$$-k_\perp^2 \tilde{A}_{||} = \frac{4\pi}{c} N_0 e \tilde{V}_{||e} \tag{23}$$

Again, magnetic perturbation and electron inertia modify the effective collision frequency:

$$0.51\nu_{ei} \to 0.51\nu_{ei} - i\omega + i\omega_*[\omega_{pe}/ck_\perp]^2 \tag{24}$$

Then the normalized dispersion relation becomes

$$(\hat{\omega} - 1)i\hat{\sigma}_{||} + \hat{\omega}^2\left(1 - \mu\frac{i\hat{\omega}}{\hat{\sigma}_\perp} + \frac{i}{\hat{\sigma}_\perp}\right) = 0 \tag{25}$$

When both effects of magnetic perturbation, $\partial A_{||}/\partial t$ and $\tilde{b}$, are included they to some extent cancel out, and the dispersion relation becomes

$$(\hat{\omega}-1)i\hat{\sigma}_{||}+\hat{\omega}^2\left(1-\mu\frac{i\hat{\omega}}{\hat{\sigma}_\perp}-\frac{i\hat{\omega}}{\hat{\sigma}_\perp}+\frac{i}{\hat{\sigma}_\perp}\right)=0 \tag{26}$$