Box Intersection

In[1031]:=
```
boxIntersection[θ_, size_, shape_, cornerRadius_] :=
  Module[{list, circleOrigin, lineEquations, edge, rx,
    ry, r, circleEquation, combinedEquation, x, y, R, cutRadius},
   If[shape == True,
    (*Circle*)
    (*Easy: Just get the point on the circle at the given angle.*)
        list = {size/2 + size/2 Cos[θ], size/2 - size/2 Sin[θ]};
    ,
        (*Square*)

    (*Calculate if the point ends up on the top edge, corner, or right edge.*)
        circleOrigin = {size - cornerRadius, 0 + cornerRadius};
        lineEquations = {size/2 + r Cos[θ], size/2 + r Sin[θ]};
        rx = Solve[0 == lineEquations[[2]], r][[1]];
        ry = Solve[size == lineEquations[[1]], r][[1]];
        edge = {(size/2 - r * Cos[θ]) /. rx, (size/2 - r * Sin[θ]) /. ry};
        If[edge[[1]] ≤ circleOrigin[[1]],
            (*On the top edge*)
            list = {edge[[1]], 0};
         ,
            If[edge[[2]] ≥ circleOrigin[[2]],
                (*On the right edge*)
                list = {size, edge[[2]]};
              ,
                (*On the corner*)
                circleEquation =
        (x - circleOrigin[[1]])^2 + (y - circleOrigin[[2]])^2 == cornerRadius^2;
                combinedEquation = circleEquation /.
         {x → size/2 + R Cos[θ], y → size/2 - R Sin[θ]};
                cutRadius = R /. Solve[combinedEquation, R][[2]];
                list = {size/2 + (cutRadius Cos[θ]), size/2 - (cutRadius Sin[θ])};
            ];
        ];
   ];
   list
  ];
```

In[441]:=
```
circleOrigin = {size - cornerRadius, 0 + cornerRadius};
lineEquations = {size/2 + r Cos[θ], size/2 + r Sin[θ]};
rx = Solve[0 == lineEquations[[2]], r][[1]];
ry = Solve[size == lineEquations[[1]], r][[1]];
edge = {(size/2 - r * Cos[θ]) /. rx, (size/2 - r * Sin[θ]) /. ry}
circleEquation = (x - circleOrigin[[1]])^2 + (y - circleOrigin[[2]])^2 == cornerRadius^2;
combinedEquation = circleEquation /. {x → size/2 + R Cos[θ], y → size/2 - R Sin[θ]};
cutRadius = R /. Solve[combinedEquation, R][[2]];
list = {size/2 + (cutRadius Cos[θ]), size/2 - (cutRadius Sin[θ])};
output = {FullSimplify[list[[1]], Assumptions → assumptions],
  FullSimplify[list[[2]], Assumptions → assumptions]}
output = output /. {size (3 + Cos[2 θ] + Sin[2 θ]) → a};
output = output /. {-2 cornerRadius (Cos[θ] + Sin[θ]) → b};
output =
  output /. {(4 cornerRadius - size) size + (-2 cornerRadius + size)^2 Sin[2 θ] → c};
output = output /. {size Cos[θ] (Cos[θ] - Sin[θ]) → d};
output = output /. {2 cornerRadius Sin[θ] (Cos[θ] + Sin[θ]) → e}
```

Out[445]=
$$\left\{\frac{\text{size}}{2}+\frac{1}{2}\,\text{size}\,\text{Cot}[\theta],\ \frac{\text{size}}{2}-\frac{1}{2}\,\text{size}\,\text{Tan}[\theta]\right\}$$

Out[450]=
$$\left\{\frac{1}{4}\left(\text{size}\,(3+\text{Cos}[2\,\theta]+\text{Sin}[2\,\theta])+2\,\text{Cos}[\theta]\left(-2\,\text{cornerRadius}\,(\text{Cos}[\theta]+\text{Sin}[\theta])+\sqrt{(4\,\text{cornerRadius}-\text{size})\,\text{size}+(-2\,\text{cornerRadius}+\text{size})^2\,\text{Sin}[2\,\theta]}\right)\right),\right.$$
$$\left.\frac{1}{2}\left(\text{size}\,\text{Cos}[\theta]\,(\text{Cos}[\theta]-\text{Sin}[\theta])+2\,\text{cornerRadius}\,\text{Sin}[\theta]\,(\text{Cos}[\theta]+\text{Sin}[\theta])-\text{Sin}[\theta]\sqrt{(4\,\text{cornerRadius}-\text{size})\,\text{size}+(-2\,\text{cornerRadius}+\text{size})^2\,\text{Sin}[2\,\theta]}\right)\right\}$$

Out[455]=
$$\left\{\frac{1}{4}\left(a+2\left(b+\sqrt{c}\right)\text{Cos}[\theta]\right),\ \frac{1}{2}\left(d+e-\sqrt{c}\,\text{Sin}[\theta]\right)\right\}$$

Middle Point

In[86]:=
```
middlePoint[size_, radius_] := Module[{},
  {size/2.0, size/2.0 + size * radius}
];
```

Short Point

In[87]:=
```
shortPoint[size_, radius_] := Module[{},
  {size/2.0 - size * radius, size/2.0}
 ];
```

Long Arm End Point

In[1359]:=
```
longArmEndPoint[θ_, size_, r_, boxIntersectionEndPoint_, middleEndPoint_] :=
 Module[{lineEquation, circleEquation, circleX, circleY, lineX, lineY, x, y},
  (*The line between the middle and long end points.*)
  lineEquation = y - middleEndPoint[[2]] ==
    (boxIntersectionEndPoint[[2]] - middleEndPoint[[2]])/
     (boxIntersectionEndPoint[[1]] - middleEndPoint[[1]]) (x - middleEndPoint[[1]]);
  (*The circle where the mid-point needs to lie.*)
  circleEquation = (x - size/2.0)^2 + (y - size/2.0)^2 == r^2;
  (*Calculate the intersection of the circle and line.*)
  circleX = NSolve[circleEquation, x][[2]];
  Print["CX: ", circleX];
  circleY = NSolve[circleEquation, y][[1]];
  Print["CY: ", circleY];
  lineY = NSolve[lineEquation /. circleX, y];
  Print["LY: ", lineY[[1]]];
  lineX = NSolve[lineEquation /. circleY, x];
  Print["LX: ", lineX[[1]]];
  Print["LA: " , {x /. lineX[[1]], y /. lineY[[1]]}];
  {x /. lineX[[1]], y /. lineY[[1]]}
 ]
```

In[1416]:=
```
Clear[x, y, x1, y1, x2, y2, lineEquation,
 circleEquation, size, r, circleX, circleY, lineY, lineX]
lineEquation = y - y1 == (y2 - y1)/(x2 - x1) (x - x1);
(*The circle where the mid-point needs to lie.*)
circleEquation = (x - size/2.0)^2 + (y - size/2.0)^2 == r^2;
(*Calculate the intersection of the circle and line.*)
circleX = NSolve[circleEquation, x][[2]];
circleY = NSolve[circleEquation, y][[1]];
lineY = NSolve[lineEquation /. circleX, y];
lineX = NSolve[lineEquation /. circleY, x];
assumptions = {size > 0, 0 < θ < π/2, x1 > x2 > 0, y1 > y2 > 0};
(*Note: The order of the solutions
  to lineX is different when computed in this case.*)
output = {FullSimplify[x /. lineX[[2]], Assumptions → assumptions],
  FullSimplify[y /. lineY[[1]], Assumptions → assumptions]}

output = output /.
   {1.` size x1^2 - 2.` size x1 x2 + 1.` size x2^2 + 1.` size x1 y1 - 1.` size x2 y1 → a1};
output = output /. {2.` x2 y1^2 - 1.` size x1 y2 + 1.` size x2 y2 -
      2.` x1 y1 y2 - 2.` x2 y1 y2 + 2.` x1 y2^2 → a2};
output = output /. {-16.` (1.` x1^2 - 2.` x1 x2 + 1.` x2^2 + 1.` y1^2 - 2.` y1 y2 + 1.` y2^2) → b};
output = output /. {r^2 (-1.` x1^2 + 2.` x1 x2 - 1.` x2^2) → c1};
output = output /. {size^2 (0.5` x1^2 - 1.` x1 x2 + 0.5` x2^2) → c2};
output = output /. {1.` x2^2 y1^2 - 2.` x1 x2 y1 y2 + 1.` x1^2 y2^2 → d1};
output = output /. {size (1.` x1 x2 y1 - 1.` x2^2 y1 - 1.` x1^2 y2 + 1.` x1 x2 y2) → d2};
output = output /. {c1 + c2 + d1 + d2 → cd};
output = output /. {x1 (4.` y1 - 4.` y2) y2 + x2 y1 (-4.` y1 + 4.` y2) → e1};
output = output /.
   {size (-2.` x1^2 + x2 (-2.` x2 + 2.` y1 - 2.` y2) + x1 (4.` x2 - 2.` y1 + 2.` y2)) → e2};
output = output /. {1.` x1^2 - 2.` x1 x2 + 1.` x2^2 + 1.` y1^2 - 2.` y1 y2 + 1.` y2^2 → f};
output = output /.
   {0.5` size x1 y1 - 0.5` size x2 y1 - 1.` x1 x2 y1 + 1.` x2^2 y1 + 0.5` size y1^2 → g1};
output = output /. {-0.5` size x1 y2 + 1.` x1^2 y2 + 0.5` size x2 y2 -
      1.` x1 x2 y2 - 1.` size y1 y2 + 0.5` size y2^2 → g2};
output = output /. {-4.` x2^2 y1 - 4.` x1^2 y2 + x1 x2 (4.` y1 + 4.` y2) → h1};
output = output /.
   {size (-2.` x1 y1 + 2.` x2 y1 - 2.` y1^2 + 2.` x1 y2 - 2.` x2 y2 + 4.` y1 y2 - 2.` y2^2) → h2};
output = output /. {r^2 (-1.` y1^2 + 2.` y1 y2 - 1.` y2^2) +
      size^2 (0.5` y1^2 - 1.` y1 y2 + 0.5` y2^2) → i};
output = output /. {size (x1 (1.` y1 - 1.` y2) y2 + x2 y1 (-1.` y1 + 1.` y2)) → j}
```

Out[1424]=
$$\left\{\frac{1}{1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2}\right.$$
$$0.5\,\left(1.\,size\,x1^2-2.\,size\,x1\,x2+1.\,size\,x2^2+1.\,size\,x1\,y1-1.\,size\,x2\,y1+\right.$$
$$2.\,x2\,y1^2-1.\,size\,x1\,y2+1.\,size\,x2\,y2-2.\,x1\,y1\,y2-2.\,x2\,y1\,y2+$$
$$2.\,x1\,y2^2+0.5\,\sqrt{\left(-16.\,\left(1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2\right)\right.}$$
$$\left(r^2\,\left(-1.\,x1^2+2.\,x1\,x2-1.\,x2^2\right)+size^2\,\left(0.5\,x1^2-1.\,x1\,x2+0.5\,x2^2\right)+\right.$$
$$1.\,x2^2\,y1^2-2.\,x1\,x2\,y1\,y2+1.\,x1^2\,y2^2+size\,\left(1.\,x1\,x2\,y1-1.\,x2^2\,y1-\right.$$
$$\left.\left.\left.1.\,x1^2\,y2+1.\,x1\,x2\,y2\right)\right)+\left(x1\,(4.\,y1-4.\,y2)\,y2+x2\,y1\,(-4.\,y1+4.\,y2)+\right.\right.$$
$$\left.\left.\left.size\,\left(-2.\,x1^2+x2\,(-2.\,x2+2.\,y1-2.\,y2)+x1\,(4.\,x2-2.\,y1+2.\,y2)\right)\right)^2\right)\right),$$
$$\frac{1}{1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2}$$
$$1.$$
$$\left(0.5\,size\,x1\,y1-0.5\,size\,x2\,y1-\right.$$
$$1.\,x1\,x2\,y1+1.\,x2^2\,y1+0.5\,size\,y1^2-$$
$$0.5\,size\,x1\,y2+1.\,x1^2\,y2+0.5\,size\,x2\,y2-$$
$$1.\,x1\,x2\,y2-1.\,size\,y1\,y2+0.5\,size\,y2^2-$$
$$0.25\,\sqrt{\left(\left(-4.\,x2^2\,y1-4.\,x1^2\,y2+x1\,x2\,(4.\,y1+4.\,y2)+\right.\right.}$$
$$\left.size\,\left(-2.\,x1\,y1+2.\,x2\,y1-2.\,y1^2+2.\,x1\,y2-2.\,x2\,y2+4.\,y1\,y2-2.\,y2^2\right)\right)^2-$$
$$16.\,\left(1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2\right)\,\left(1.\,x2^2\,y1^2-2.\,x1\,x2\,y1\,y2+\right.$$
$$\left.\left.\left.1.\,x1^2\,y2^2+r^2\,\left(-1.\,y1^2+2.\,y1\,y2-1.\,y2^2\right)+size^2\,\left(0.5\,y1^2-1.\,y1\,y2+0.5\,y2^2\right)+\right.\right.\right.$$
$$\left.\left.\left.size\,(x1\,(1.\,y1-1.\,y2)\,y2+x2\,y1\,(-1.\,y1+1.\,y2))\right)\right)\right)\Big\}$$

Out[1441]=
$$\left\{\frac{0.5\,\left(a1+a2+0.5\,\sqrt{b\,cd+(e1+e2)^2}\right)}{f},\ \frac{1.\,\left(g1+g2-0.25\,\sqrt{(h1+h2)^2+b\,\left(d1+i+j\right)}\right)}{f}\right\}$$

Testing

In[1442]:=
```
Manipulate[
 angle = θ π / 180;
 Graphics[
  {
   Thick, Black,
   If[circle,
    Circle[{size / 2.0, size / 2.0}, size / 2.0],
    Rectangle[{0, 0}, {size, size}, RoundingRadius → cornerRadius * size]
   ],
   PointSize[Medium],
   Point[{size / 2.0, size / 2.0}],
   Red, Point[boxIntersection[angle, size, circle, size * cornerRadius]],
   Green, Point[middlePoint[size, .135]],
   Blue, Point[shortPoint[size, .185]],
   Orange, Point[longArmEndPoint[angle, size, size .33, boxIntersection[
      angle, size, circle, size * cornerRadius], middlePoint[size, .135]]]
  },
  PlotRange → All,
  Axes → True
 ]

 , {{θ, 36}, 0, 90, 0.1}, {size, 1, 100, 1},
 {circle, {True, False}}, {cornerRadius, 0, 0.5, 0.01}]
```

Out[1442]=