

# Foundations of Deep Learning



ALF

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# Convolutional Neural Nets

Exploiting stationarity, locality, and compositionality of natural data

# Signals can be represented as vectors



$$\mathbf{x} = [x_1 \ x_2 \ x_3 \ \dots \ x_t \ \dots]^\top$$

$x_t$  are waveform heights



$$\mathbf{x} = [x_{11} \ x_{12} \ \dots \ x_{1n} \ x_{21} \ x_{22} \ \dots]^\top$$

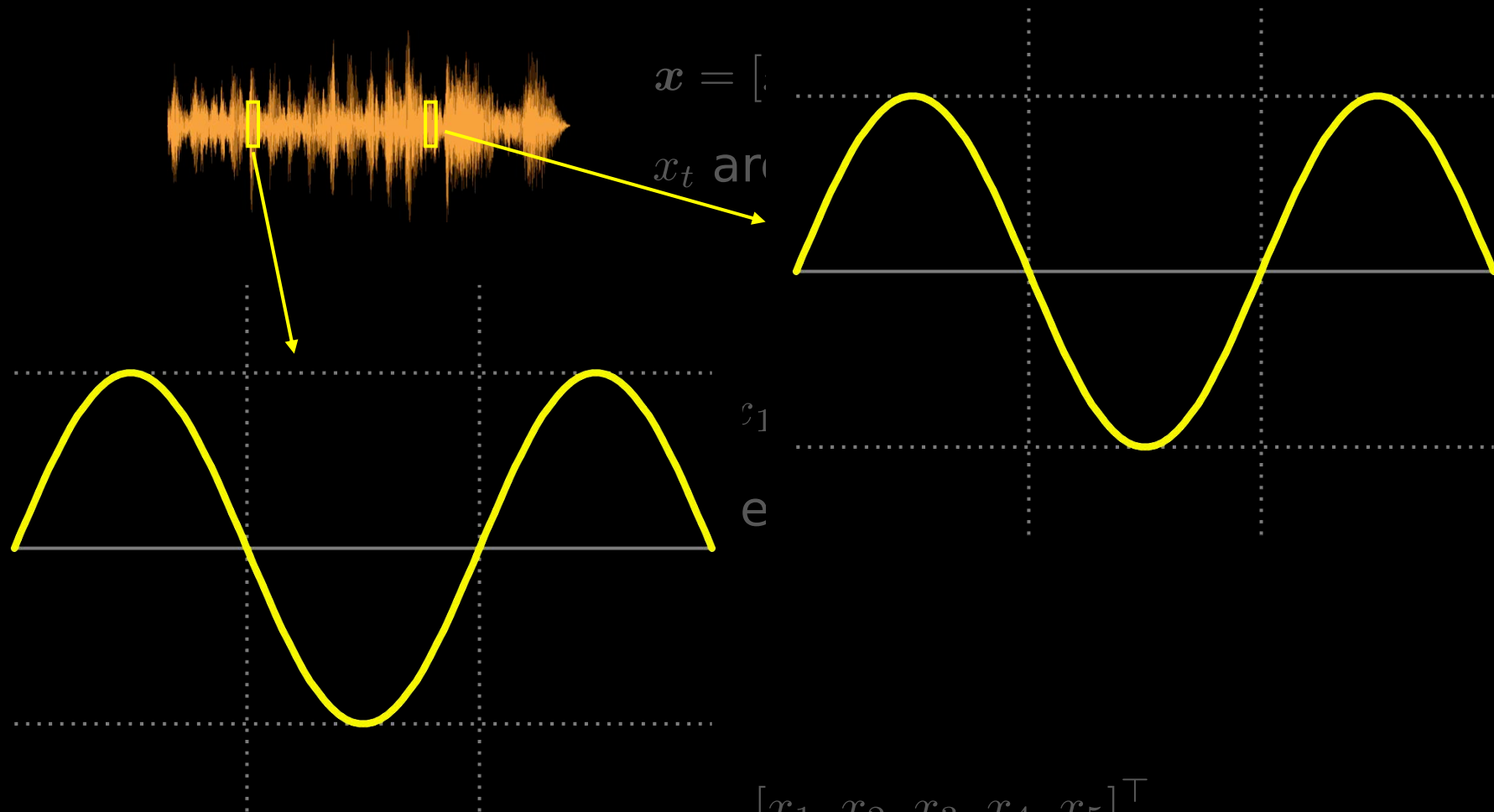
$x_{ij}$  are pixel values

“John picked up the apple”

$$\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^\top$$

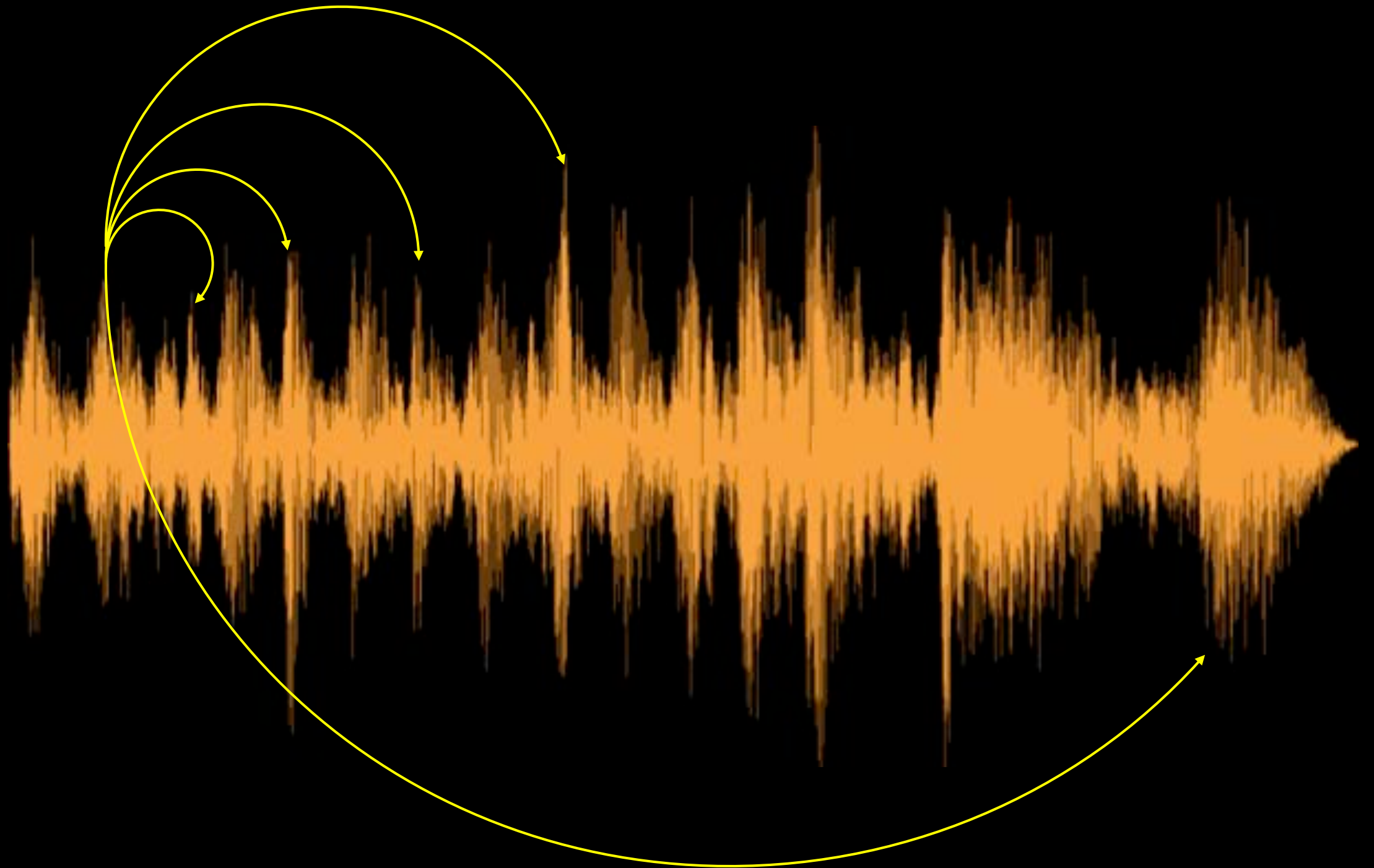
$x_t$  are one-hot vectors

# Signals can be represented as vectors



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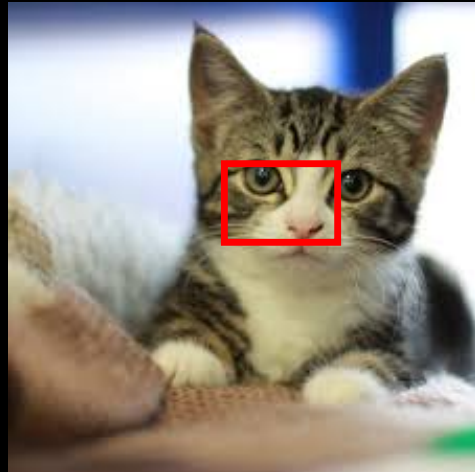


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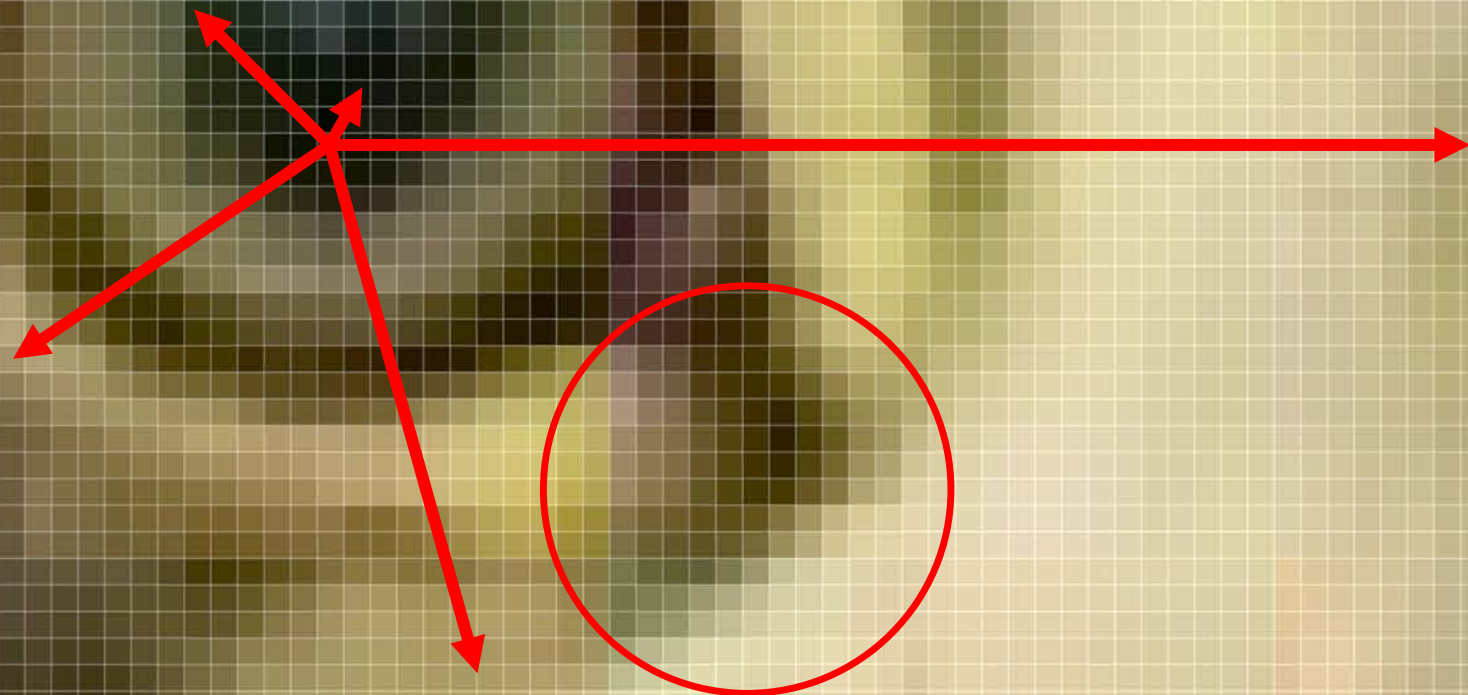
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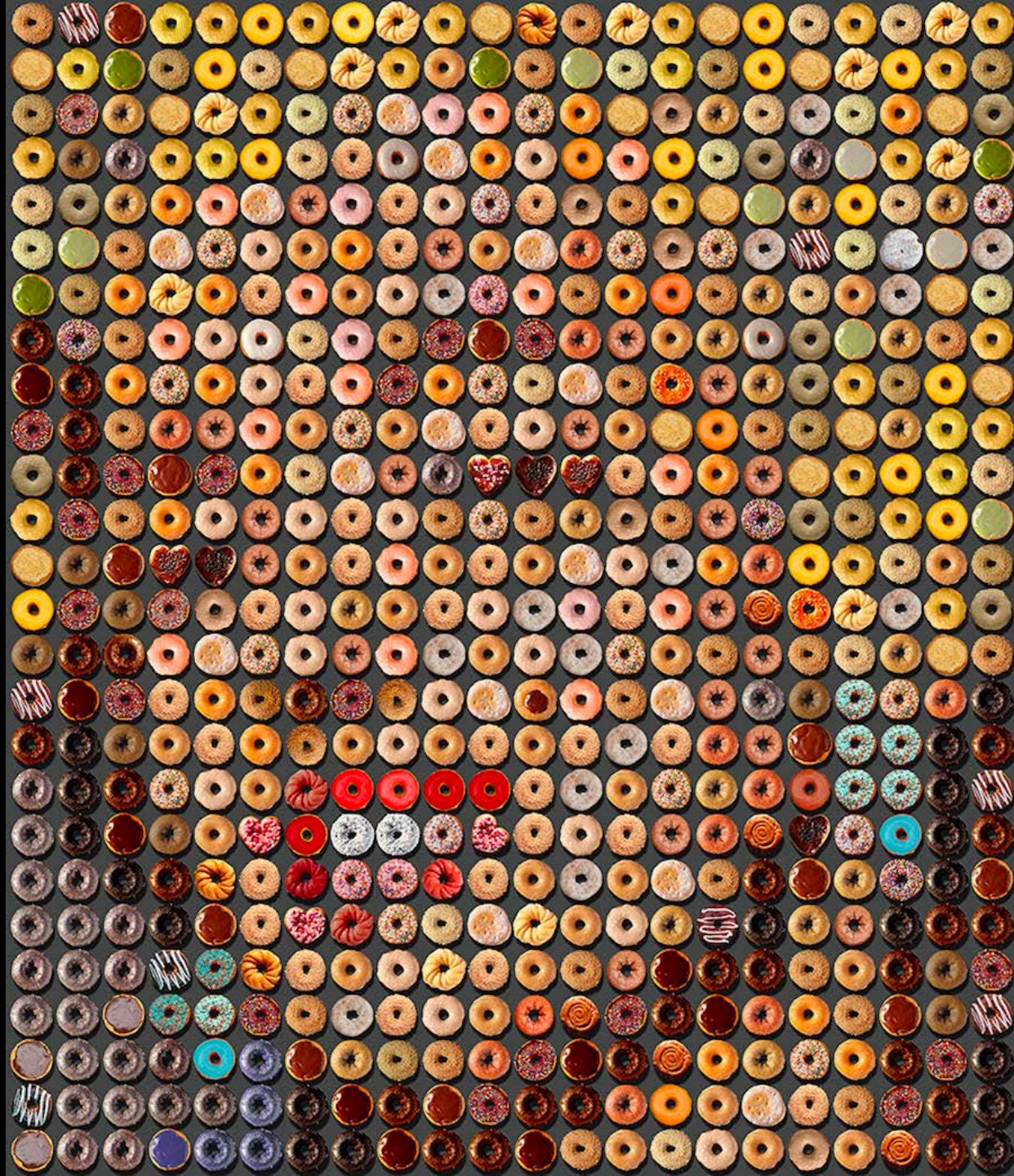
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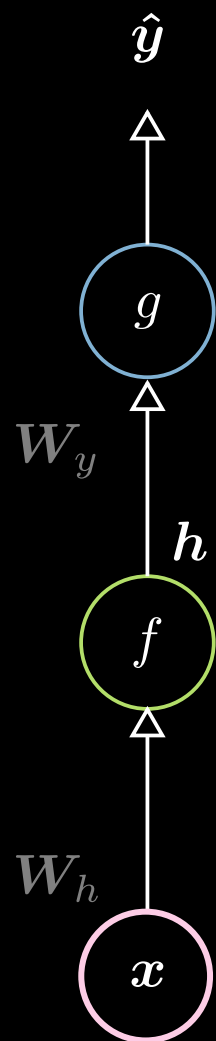








# Fully connected (FC) layer



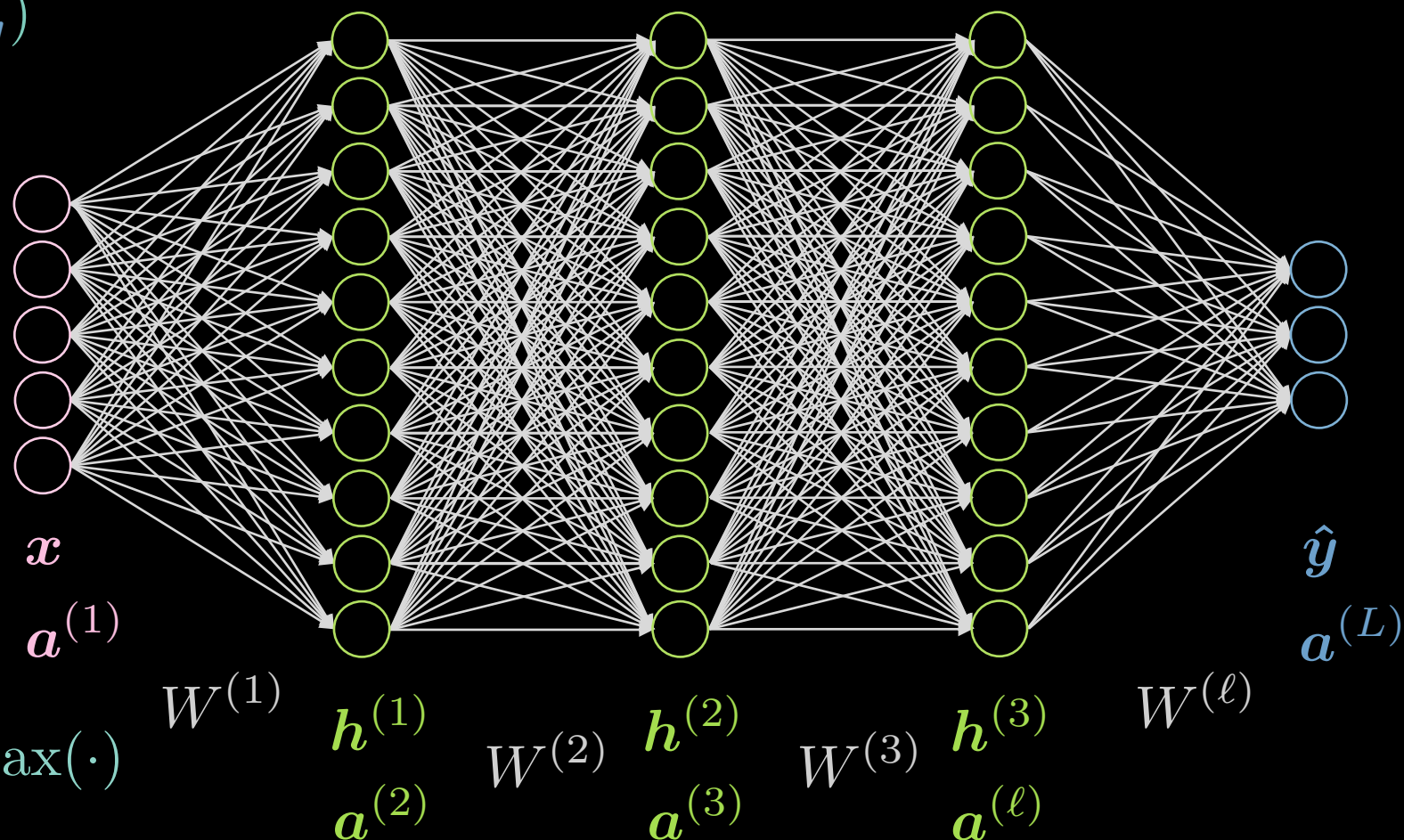
$$h = f(W_h x + b_h)$$

$$\hat{y} = g(W_y h + b_y)$$

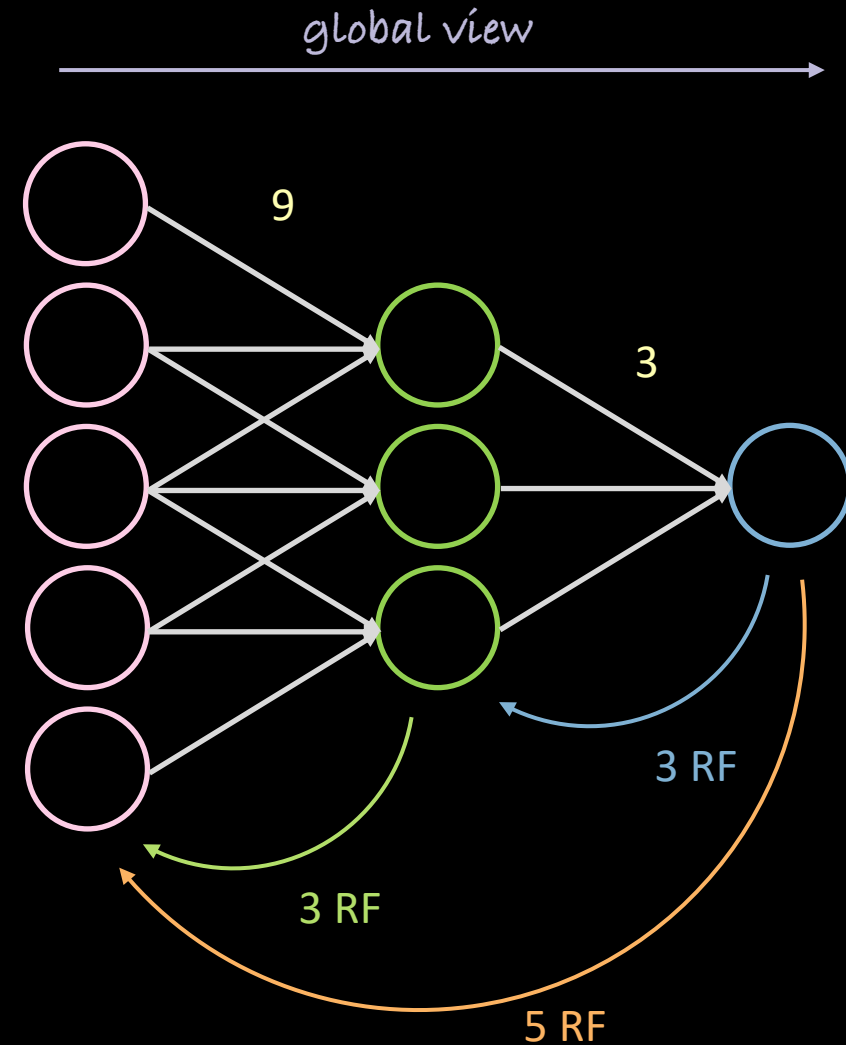
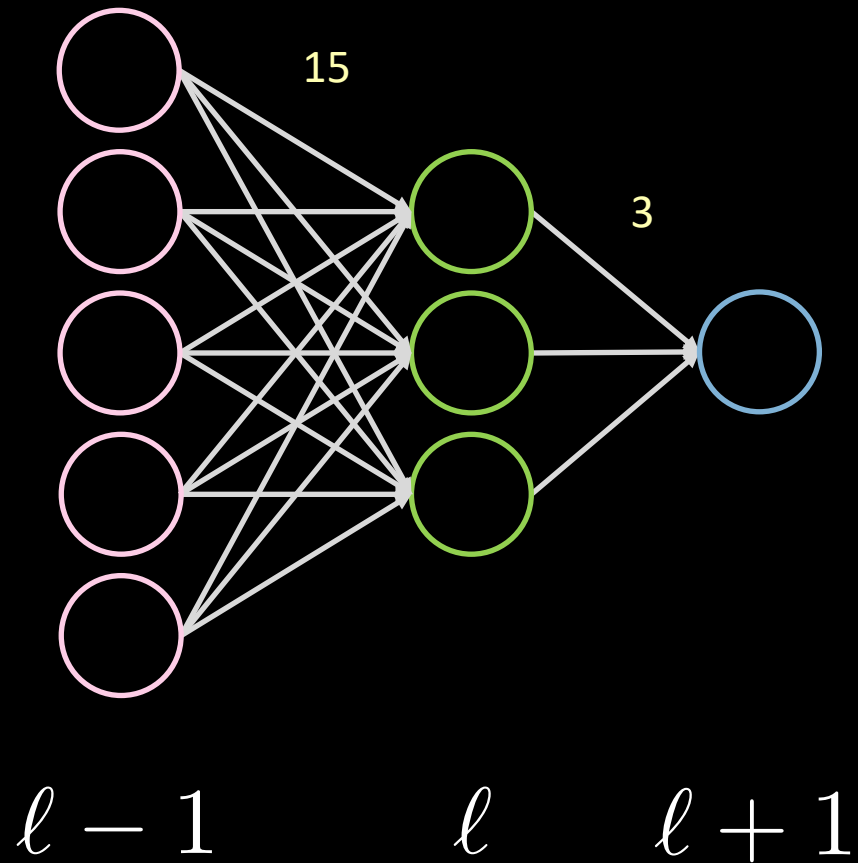
$f, g = (\cdot)^+, \sigma(\cdot), \tanh(\cdot), \text{soft}(\text{arg})\max(\cdot)$

$j$ -th row of  $W^{(1)}$

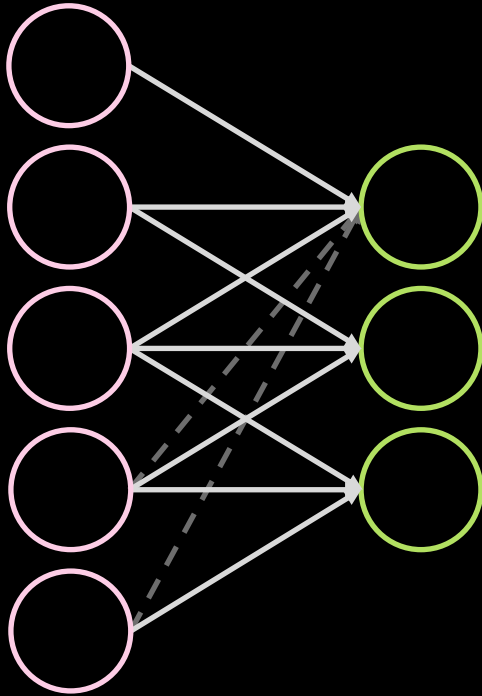
$$a_j^{(2)} = f(\boxed{w^{(j)}} x + b_j) = f\left(\left(\sum_{i=1}^n w_i^{(j)} x_i\right) + b_j\right)$$



Locality  $\Rightarrow$  sparsity



# Stationarity $\Rightarrow$ parameters sharing

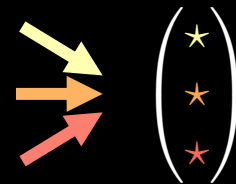
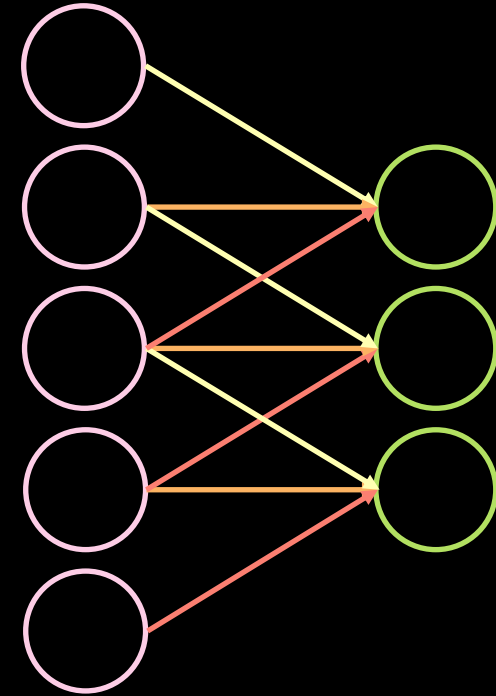


## Parameters sharing

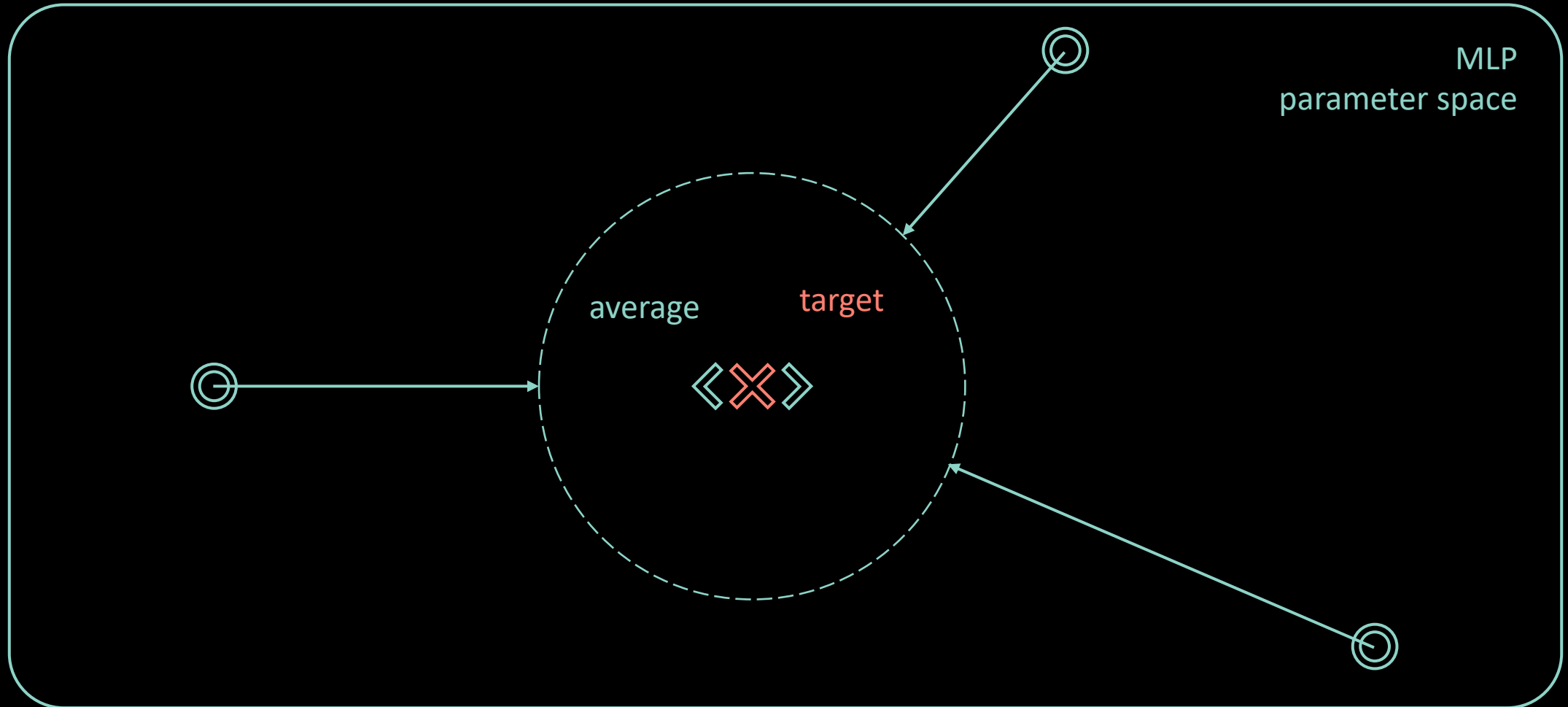
- faster convergence
- better generalisation
- not constrained to input size
- kernel independence  
 $\Rightarrow$  high parallelisation

## Connection sparsity

- reduced amount of computation

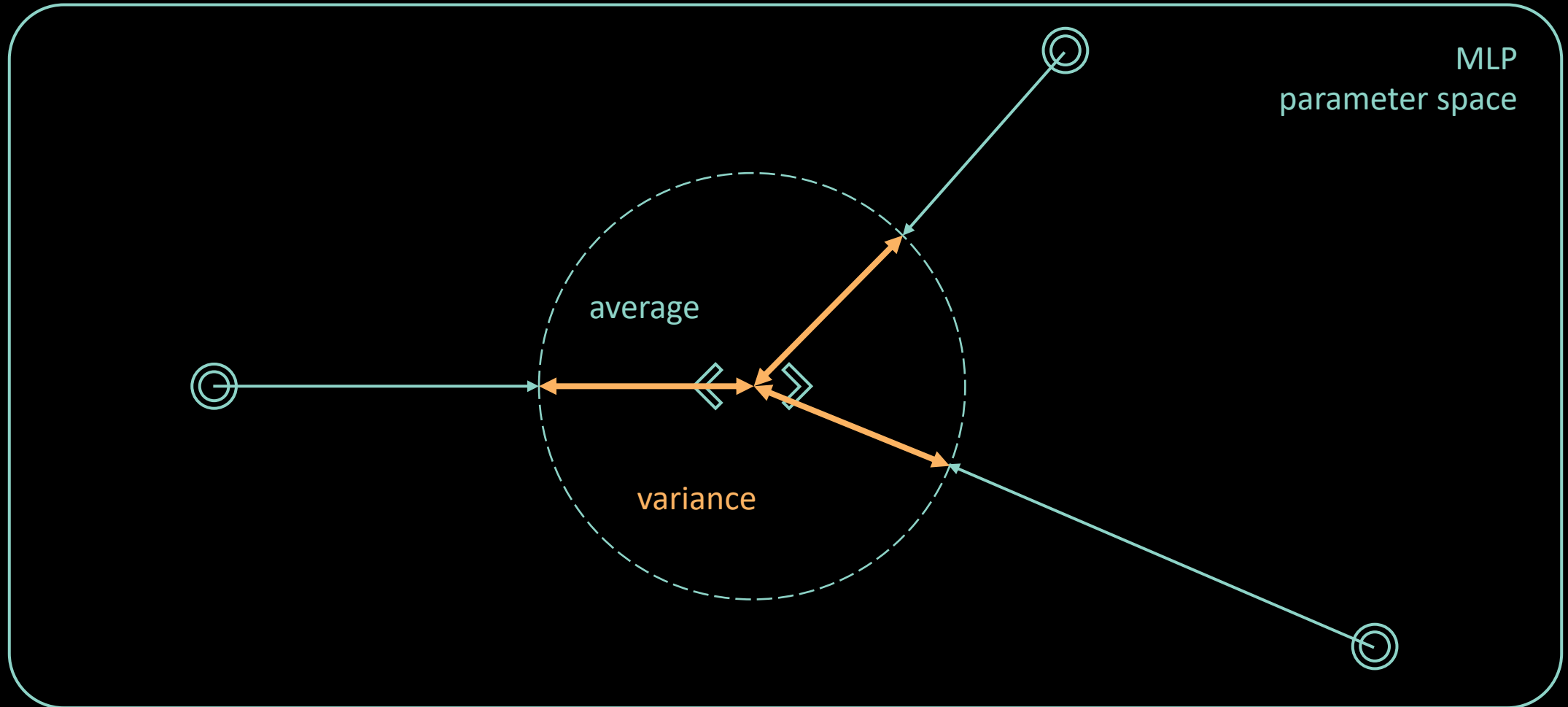


# Generalisation error reduction

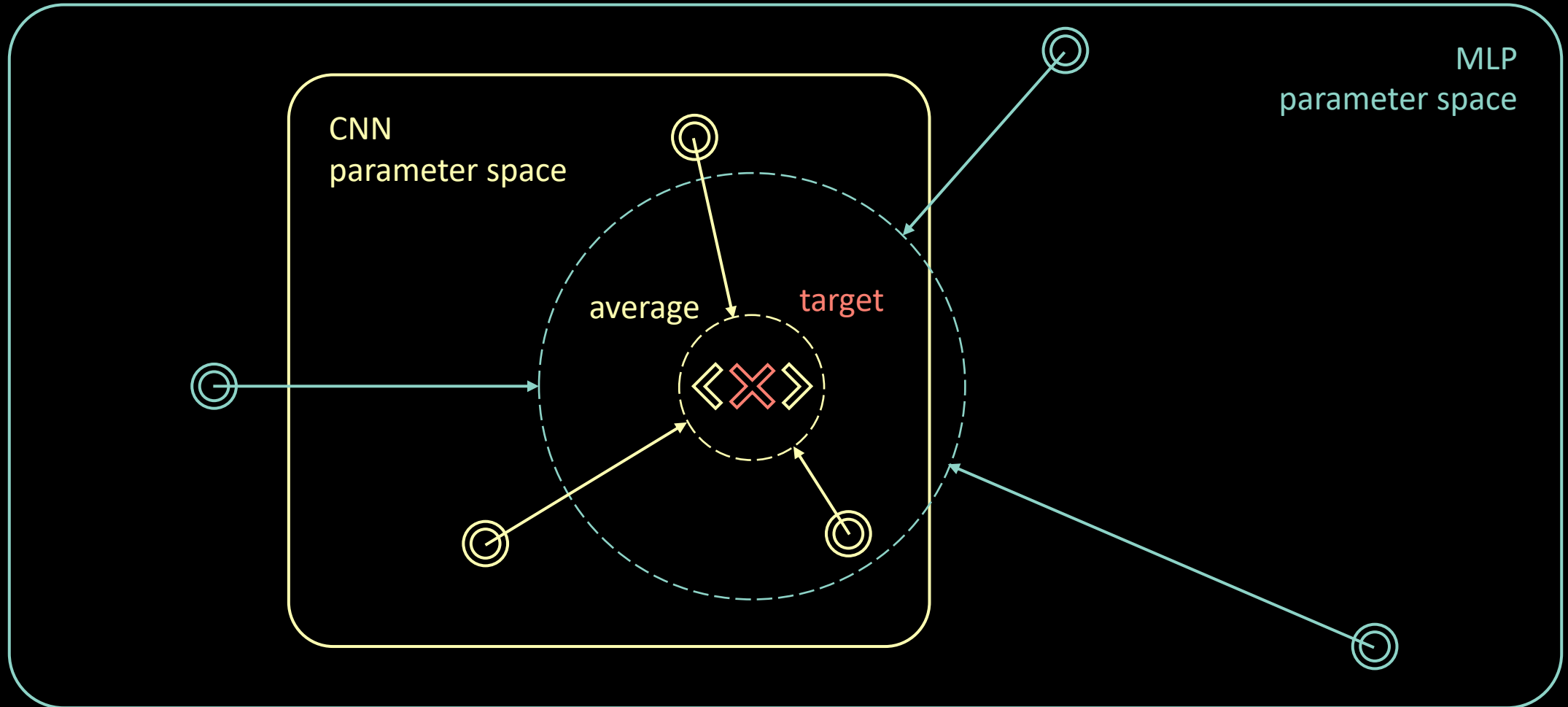




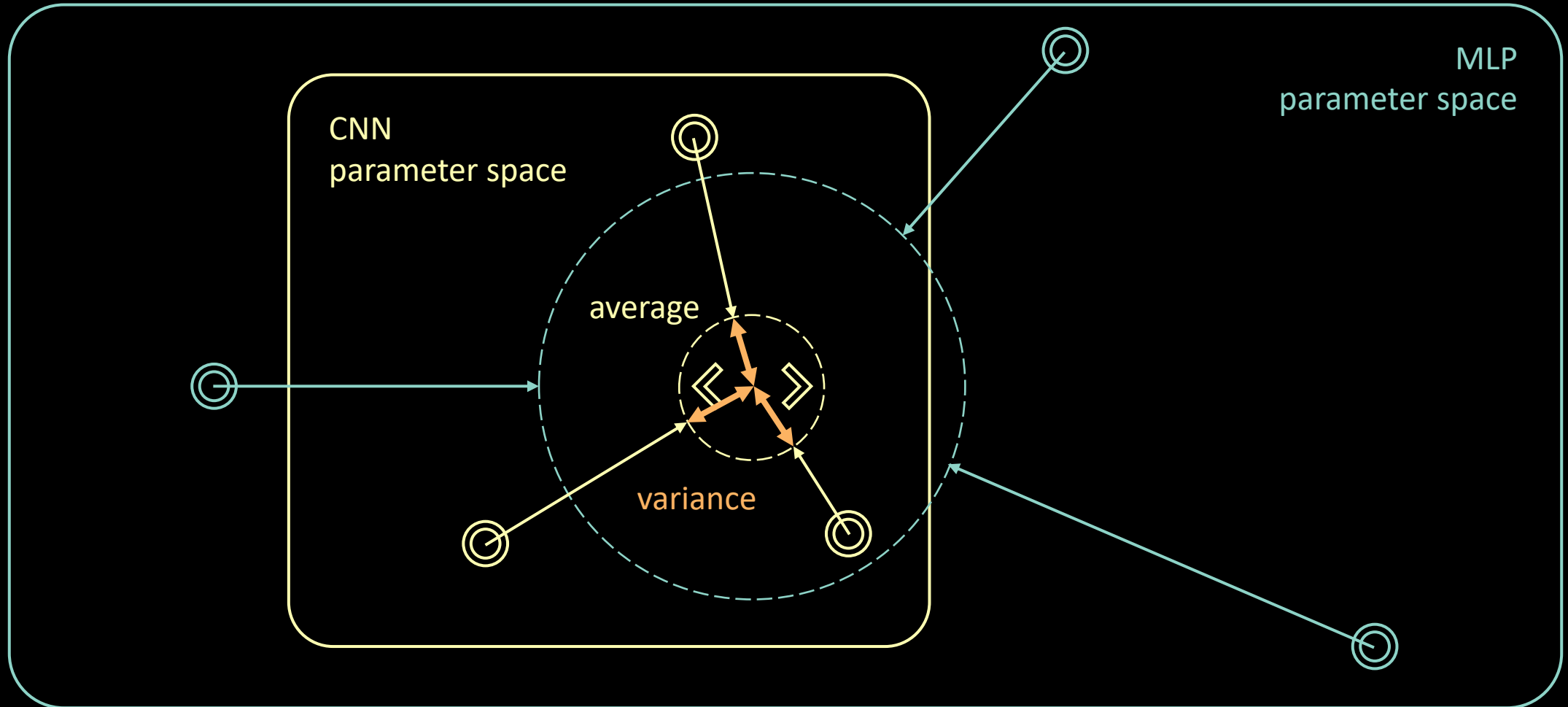
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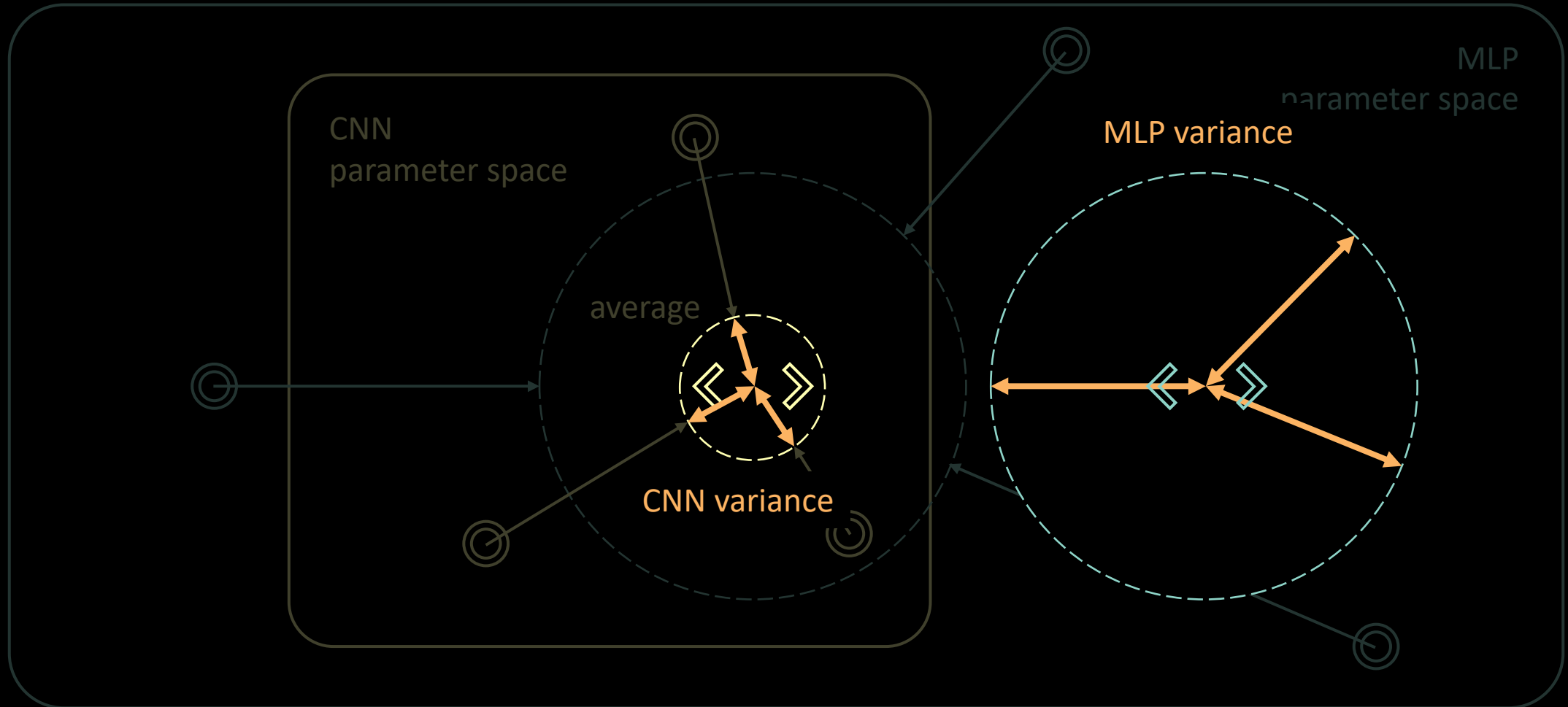
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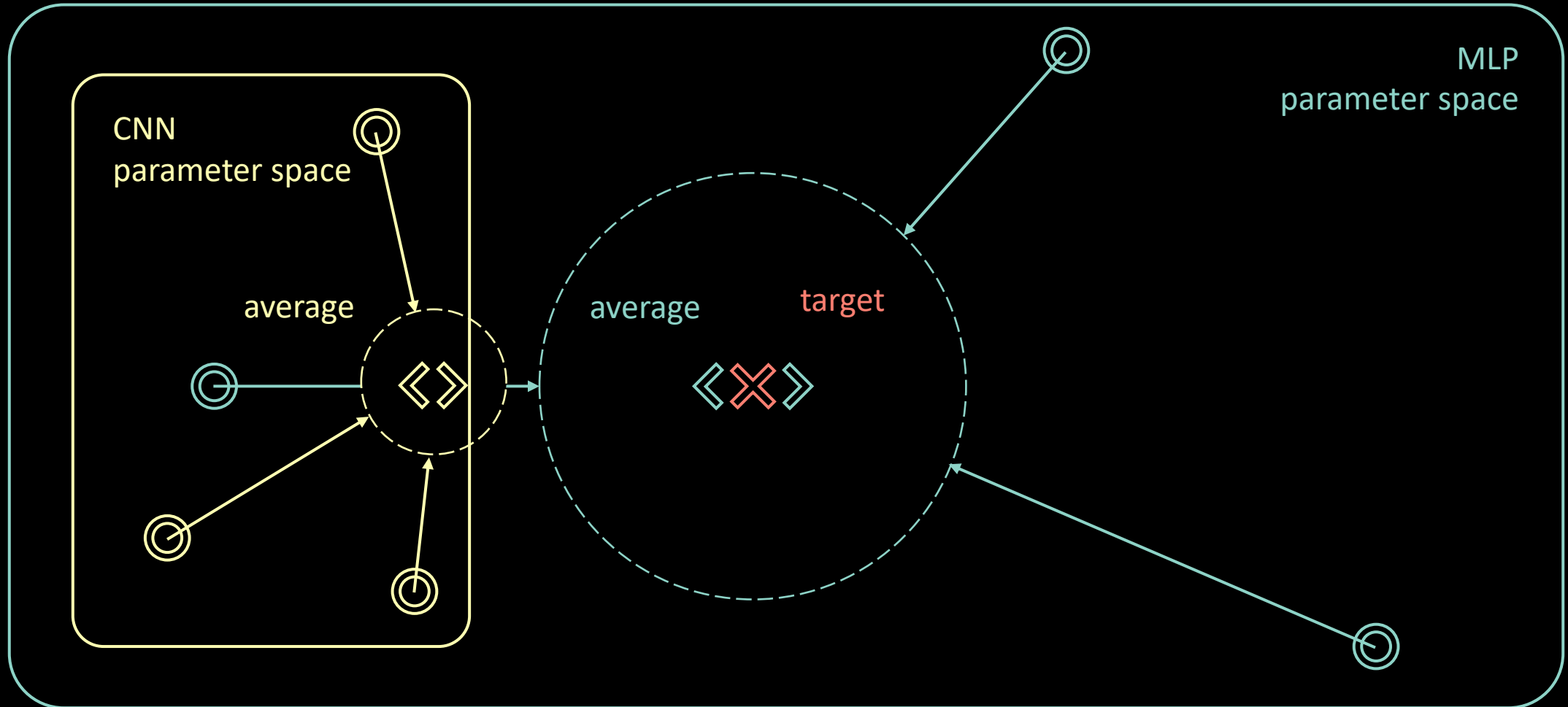


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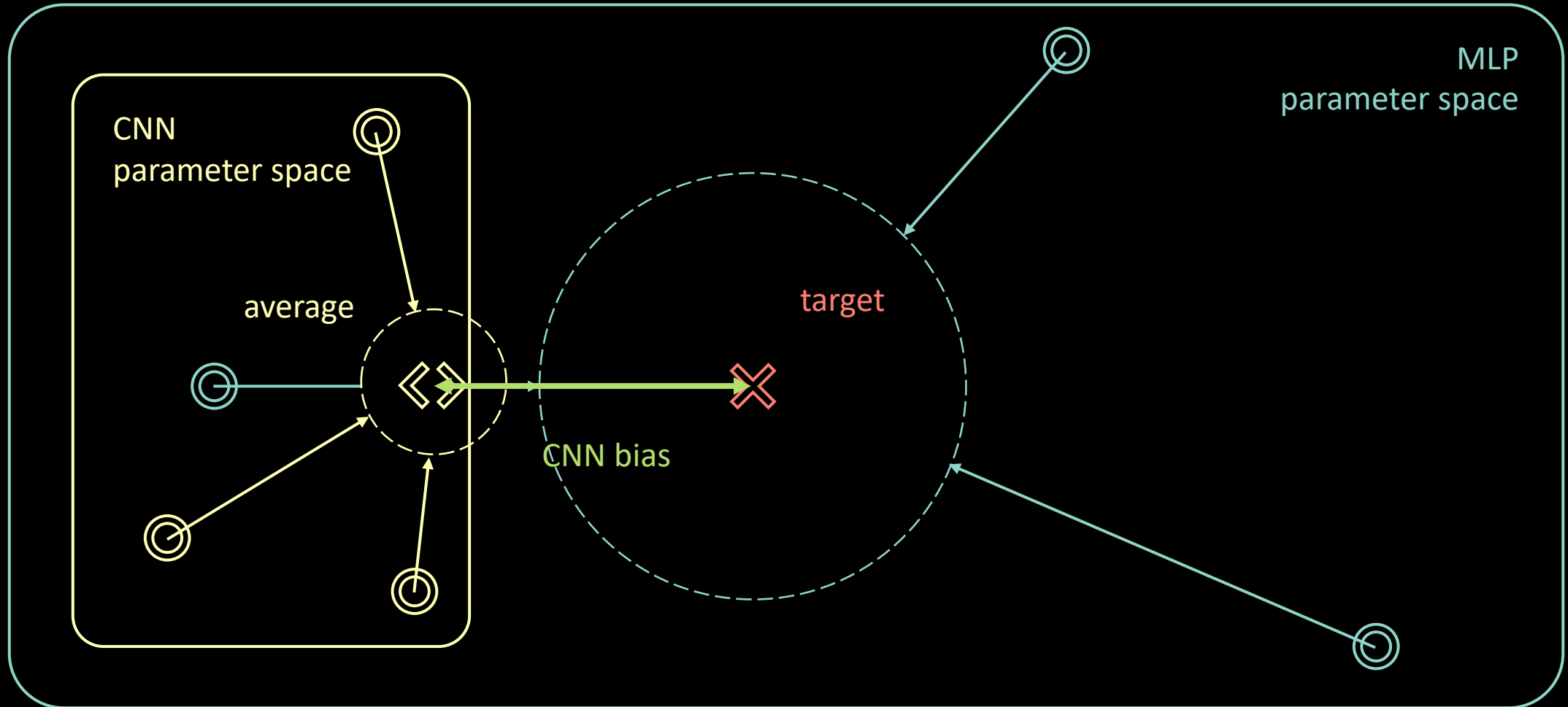




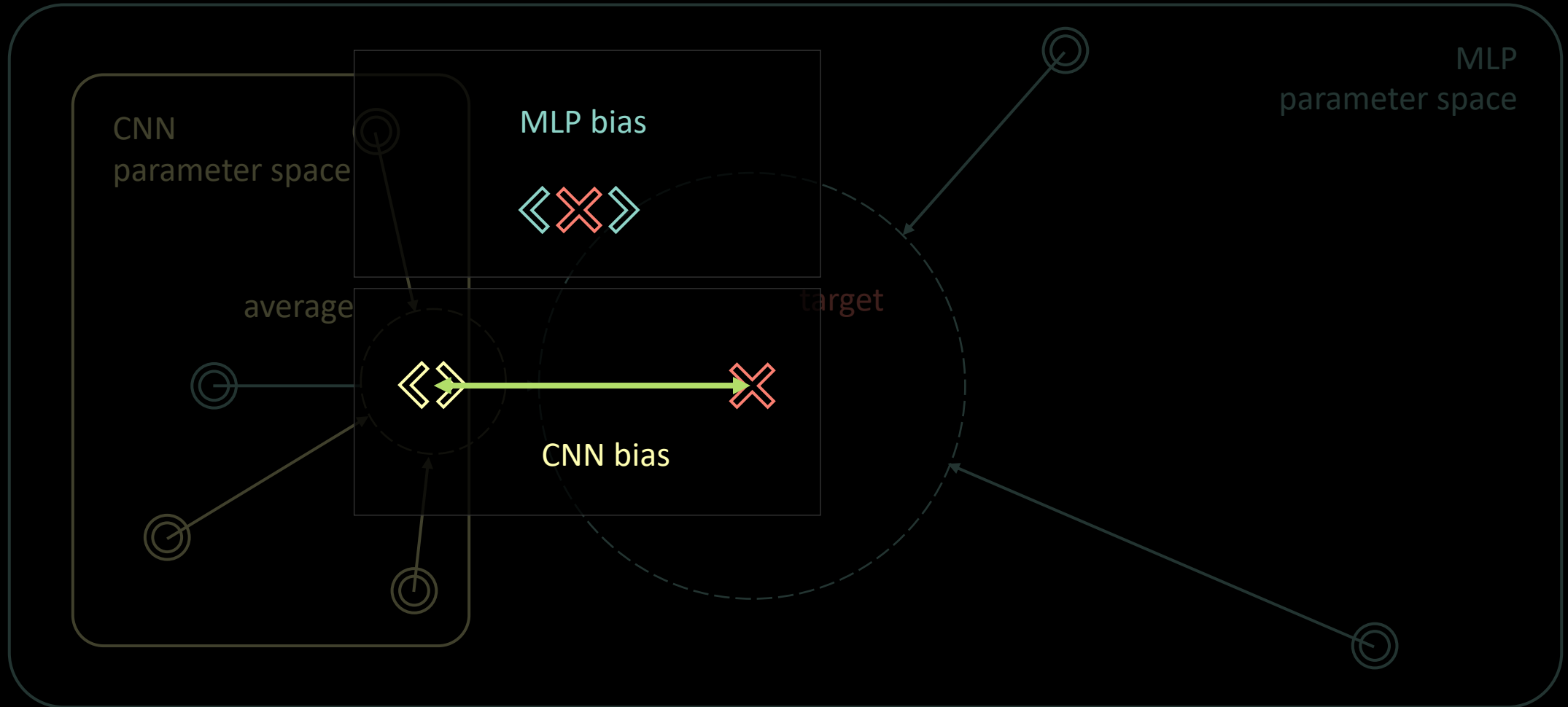
# Misspecification of model constraints



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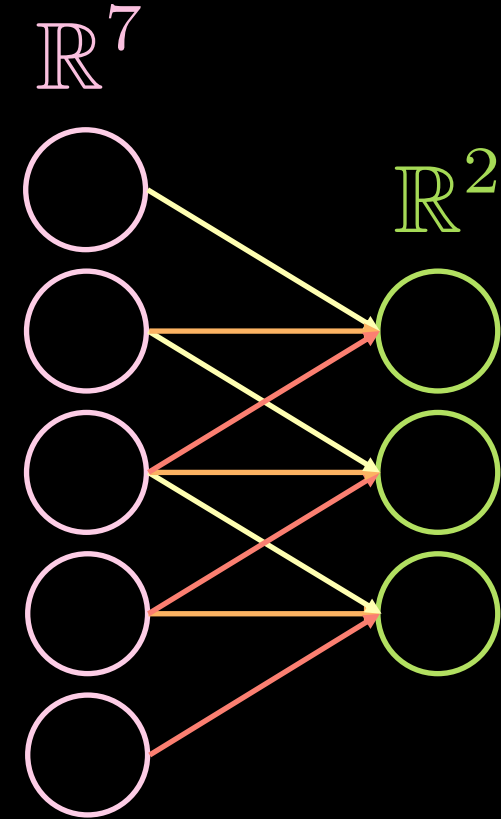
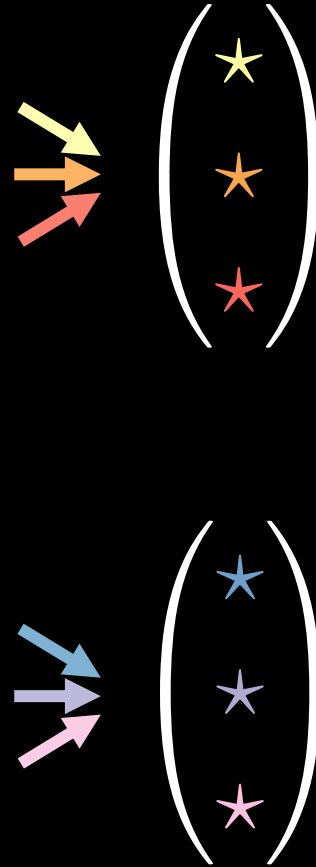
# Misspecification of model constraints



# Kernels – 1D data

kernel size:  $2 \times 7 \times 3$

1D data uses 3D kernels-collection!

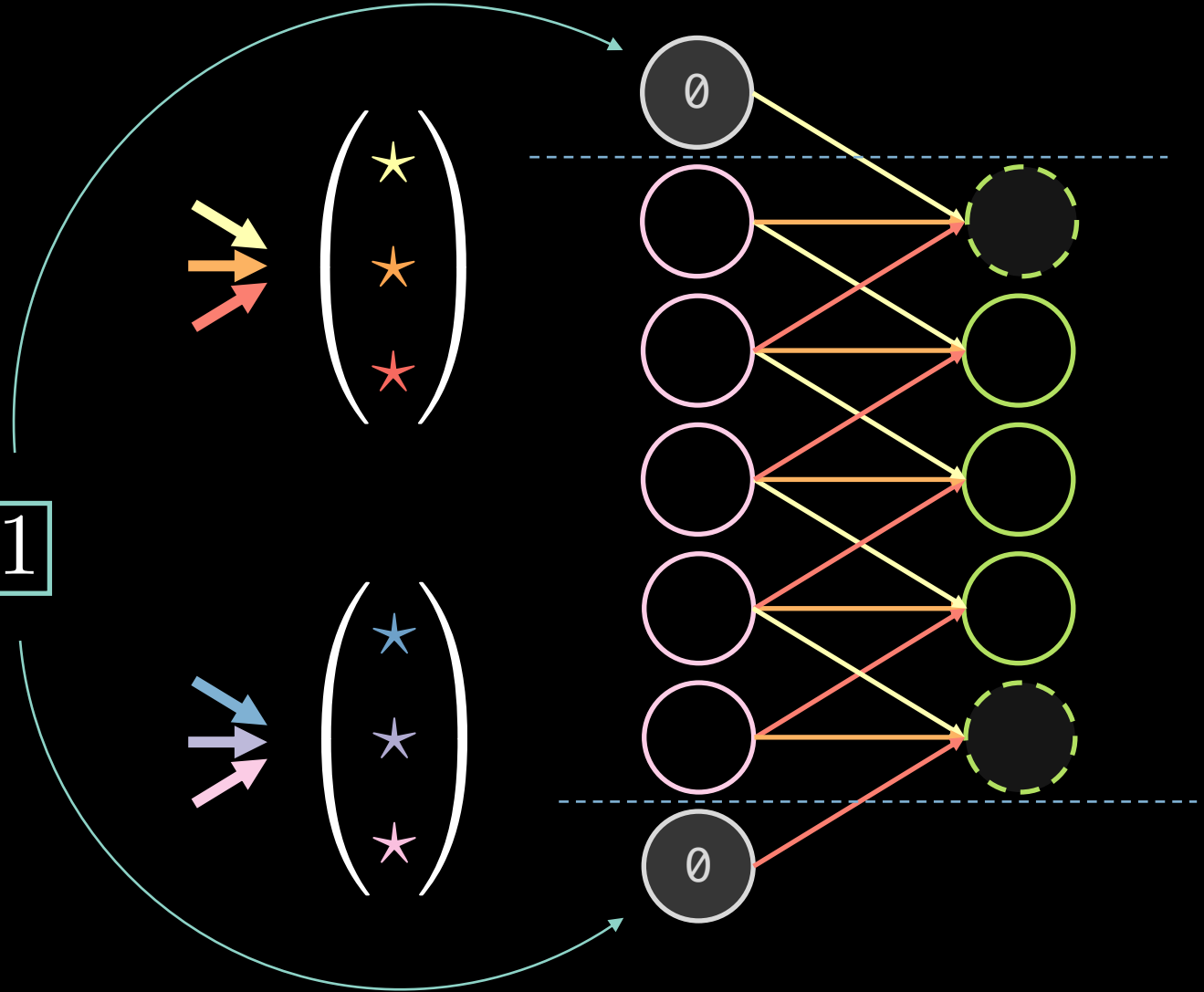




# Padding – 1D data

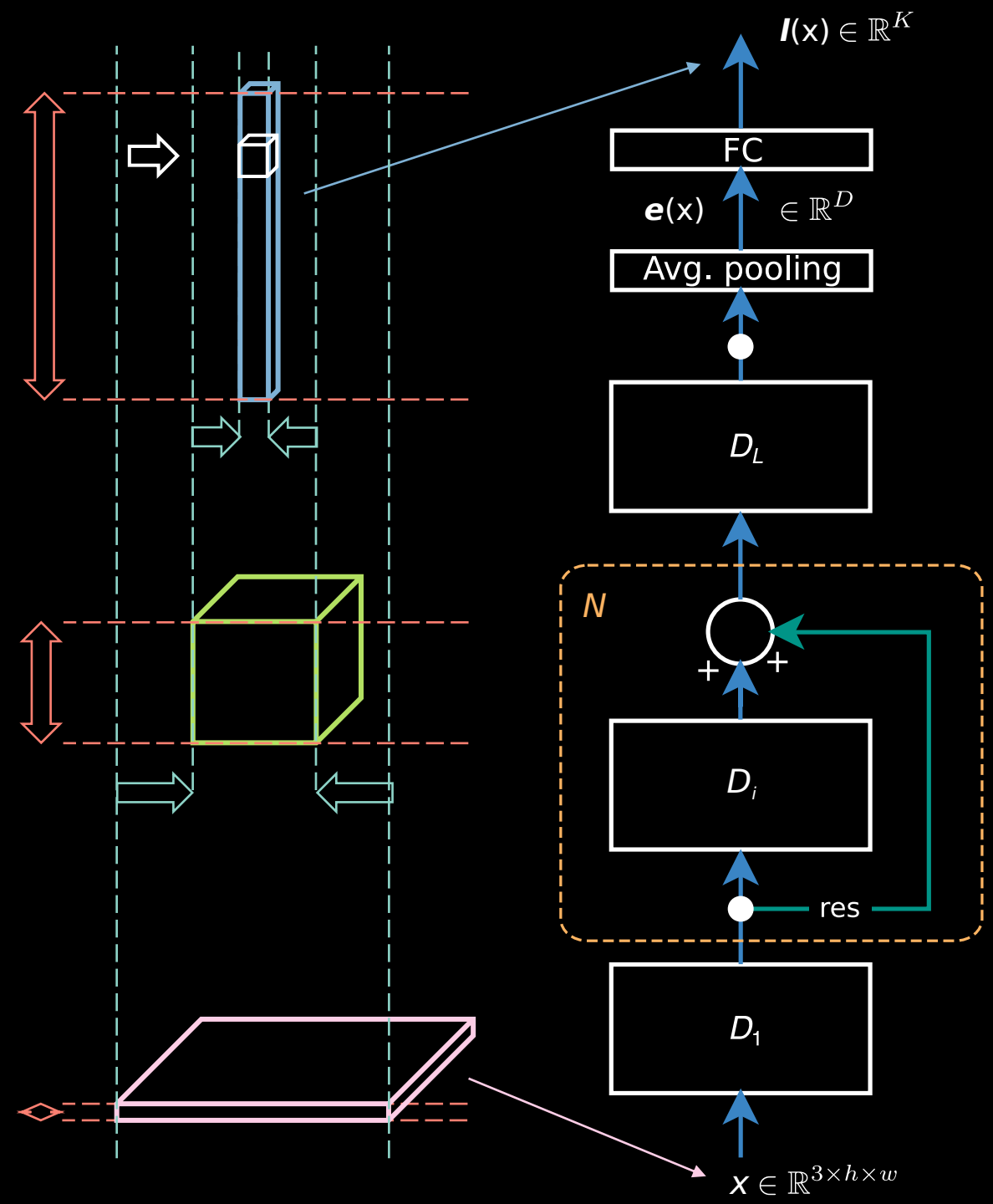
kernel size:  $2 \times 7 \times 3$

zero padding:  $(3 - 1)/2 = 1$



# Standard spatial CNN

- Multiple layers
  - Convolution
  - Non-linearity (ReLU and Leaky)
  - Pooling
  - Batch normalisation
- Residual bypass connection



# Pooling

$$\|x\|_p := \left( \sum_i |x_i|^p \right)^{1/p}$$

$$\|x\|_p \rightarrow \max(x), p \rightarrow +\infty$$

